## Department of Business Administration and Economics

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Quantifying know ledge on consumers' payment behavior in retailing

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#### Abstract

This paper shows how information about payments contained in normal market basket data can be used by retailers. A possible structure of hypotheses explaining consumers' payment behavior is described and a sample hypothesis is created and tested with actual market basket data. Knowledge of this kind can be used by retailers to better predict the amounts of cash needed to make change which in turn could help to minimize the costs of cash management. Results of payment analysis can also be used to develop and manage new features of promotions. Examples are given for both kinds of possible uses.


## 1. The problem of cash in retailing

Probably somewhat ironically a professor of mathematics from Canada recently suggested that the USA should introduce an 18 -cent coin (Shallit 2003). This would reduce the average number of coins necessary for change at retailing cash desks, he argues. Already in 1995 another author had come to the same result (Young 1995).

A reduction of change could be advantageous for everybody: Buyers would not need to purge their purses of small coins so often. For Young this is the most important problem of change money (Young 1995: 1). In comparison to the USA, where banknotes from 100 cent upwards exist, this problem seems to be even more pressing in Europe because here the smallest banknote is worth 500 cent. Retailers would save time and money because counting and collecting coins from cash desks could be achieved faster and fewer heavy coins would have to be transported to banks.

Both Young and Shallit base their recommendations on the assumption that each amount of change below the denomination of the smallest banknote has the same probability, i. e. happens just as often as any other amount (Young 1995: 23; Shallit 2003: 1). Shallit even acknowledges that this assumption may not be realistic in retailing because the high number of prices ending with the digit 9 could have an effect on the amounts of change (Stiving/Winer 1997 report the findings of an empirical study of this phenomenon and give a number of further references for this topic). Nevertheless Shallit sticks to the assumption because he cannot come up with a better hypothesis. For him this not a problem because in his paper he is mostly concerned with questions of computability and only uses the business environment to explain his problem. Apparently this has not been noticed very often in the reactions which were triggered by Shallit's article (for references to newspaper and online articles see www.math.uwaterloo.ca/~shallit/ papers.html): The provocative title of the article has obviously paid off.

The present paper is based on real transaction data from a German retailing chain. In addition to food and beverages this chain also offers textiles, toys, household appliances and a number of other product categories. Pertaining to article prices these baskets can be seen as representative for a big share of German retailing. The amount of change needed in a transaction is determined by two factors, firstly the basket sum which is the amount due for a market basket, and secondly the payment sum which is the amount of cash the buyer originally uses to settle the

Recommending new coins

Fewer change advantageous for everybody

Unrealistic assumptions

Retailers have payment information in market basket data

Potentials of payment analysis

Organization of the paper
demanded basket sum and thus at least equals the basket sum. The frequencies of different basket sums in the database will be used to show that Young's and SHALLIT's assumption of equally distributed change amounts is far from being realistic.

The data additionally shows how buyers have actually paid, i. e. which amounts of cash they gave in reaction to which basket sums. For example, to a basket sum of 1.45 Euro a buyer could have reacted with a payment sum of 2.00 Euro which would have determined a change amount of 0.55 Euro. This information can be used to evaluate hypotheses on buyers paying behavior by comparing the distribution of change amounts it predicts with the actual distribution of change amounts in the data. Building on this methodology a way to use transaction data to automatically evaluate such hypotheses will be explained in general terms.

The most important way for retailers to use payment information from transaction data regards rationalization. The way cashiers do their job can be optimized. For example cash desks could automatically support cashiers more than they do now. Different ways to improve the process of cashing will be discussed. But this kind of information can also be important for operational decisions. For example, retailers might be able, by changing the prices of certain articles, to induce such basket sums which would decrease the amount of cash needed for change making. This question will be considered theoretically. Another way for retailers to use information about payments not just for optimizing processes but in connection with selling involve special promotions which are directly connected with basket sums or change amounts. Some possibilities for this will also be sketched out.

The remainder of the paper is organized as follows: After a short explanation of the market basket database in the following section 2 section 3 defines the cash making problem formally as a mathematical foundation of payment analysis. In section 4 results of a payment analysis of real transaction data will be presented and a specific hypothesis on buyers paying behavior will be developed and tested. Section 5 gives recommendations on how retailers could try to use payment analysis. Aspects are distinguished as regarding rationalization on the one and new ways of promotions or other ways to reinforce sales on the other hand. Section 6 gives a short summary of the paper.

## 2. The market basket database used

All in all the database of market baskets used for the present research contains information on more than 1.5 million market baskets. The database contains the complete market baskets of four outlets of a medium sized German retailing company. The baskets were collected during a period of six months from November 2002 to April 2003. But since analysis in the following sections will only be concerned with the paying behavior of retail customers in general the baskets will not be distinguished by outlet or month. From each basket only the basket sum, the payment sum and the change amount, which can be inferred from the first two variables, are needed for the analysis. Basically the following enquiry rests on tables containing the frequencies of specific basket and payment sums. The technical question how the actual values in the cells of these tables are to be extracted from the market basket database will not be addressed in detail.

## 3. A formal representation of the change making problem

To clarify the problems on which the following empirical investigation is based this section formally states the change making problem. The terminology closely follows that used by Wright 1975 and Shallit 2003.

Let $D$ different denominations for coins in a coin system (of course the following analysis applies to banknotes as well but banknotes are not specifically mentioned to keep the problem statement simple) be $n_{1}<n_{2}<\ldots<n_{\text {D }}$. For the cashier each denomination shall be available in unrestricted numbers. Also $n_{1}=1$ (cent) should hold to make sure that any amount of money can be composed in the given coin system. All the other $n_{\mathrm{i}}$ are given as multiple amounts of $n_{1}$.

With these symbols the optimal representation problem is to find for a given positive $N$ such coefficients $m_{\mathrm{i}}$ so that $N=\sum_{i} 1 \leq i \leq D m_{i} n_{i}$ holds and $Z=\sum_{i} 1 \leq i \leq D$ with $m_{\mathrm{i}} \geq 0$, the number of coins necessary to compose $N$, is minimized. The optimal $Z$ for a specific $N$, which depends on the given denominations $n_{\mathrm{i}}$ (the coin system) is written as $Z=\operatorname{opt}\left(N ; n_{1}, n_{2}, \ldots, n_{D}\right)$.

The optimal representation problem comes up millions of times every day at cash desks in retailing outlets. It is probably not always solved satisfactorily although this is unproblematic with the current Euro coin system (1, 2, 5, 10, 20, 50, 100, 200 cent coins and $10.00,20.00,50.00,100.00,200.00$ and 500.00 Euro banknotes). A possible algorithm for this task is the following: To compose a sum $N$ choose the coin with the highest $n<N$ so often ( $x$ times) as ( $x \cdot n$ ) $<N$ holds. After that, substitute $N$ with $N-(x \cdot n)$ and start again with the first step. Not all possible sets of denominations have the property of making this greedy (the name refers to the fact that always the highest possible coin is taken next) algorithm work: For example, if only the coins $(10,6,1)$ existed, for a sum of $N=12$ the algorithm described above would give three coins ( $1 \times 10,2 \times 1$ ) although ( $2 \times 6$ ) would work with one coin less.

No polynomial-time algorithm is know for answering the question whether the greedy algorithm could compose a specific sum $N$ given a specific set of denominations with the smallest possible number of coins (Kozen/Zaks 1994 prove that for this question there is no better method than the enumerative one). Only a polynomial-time algorithm exists that shows whether in a specific set of denominations every possible sum can be successfully composed with the greedy

Change making problem

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algorithm (Chang/Gill 1970; Pearson 1994; Verma/Xu 1994). But with certain boundaries for the highest possible denominations and the highest possible number of denominations theoretical problems of this kind do not hinder a practical calculation of results: Even normal personal computers are fast enough to do enumerative calculations in practical amounts of time. Albeit, this problem is of minor importance because for a given coin system it can always be known whether the greedy algorithm works and coin systems do not tend to change very fast.

Optimal denomination problem

The discussion has already touched the optimal denomination problem: Which denominations minimize the number of coins necessary for making change? This aks for those $n_{\mathrm{i}}$ that minimize the average number of coins needed to compose change amounts up to a Limit of $L$ :

$$
\operatorname{cost}\left(L ; n_{1}, n_{2}, \ldots, n_{D}\right)=\frac{1}{L} \sum_{0 \leq i \leq L} \operatorname{opt}\left(i ; n_{1}, n_{2}, \ldots, n_{D}\right)
$$

In theory, the optimal denomination problem has been dealt with in several papers, pioneered by Hentsch (e. g. Hentsch 1973, 1975; Caianiello/Scarpetta/Simoncelli 1982; Cramer 1983; Sumner 1983; Van Hove 2001).

## 4. A practical example of payment analysis

For an introduction into payment analysis of real market baskets Figure 1 shows the cumulated frequencies of all basket sums encountered in the data.


Fig. 1: Empirical distribution of payment sums

50 \% of all basket sums are below 12.70 Euro, more than 90 \% are below 45 Euro
Basket sum frequencies and more than 98.9 \% are below 100 Euro. Figure 1 does not show basket sums above this amount because these are very scarce.

| Basket sum | Relative Frequency |
| ---: | ---: |
| $5.99 €$ | $0.4869 \%$ |
| $0.99 €$ | $0.4544 \%$ |
| $1.99 €$ | $0.4273 \%$ |
| $4.99 €$ | $0.3756 \%$ |
| $9.99 €$ | $0.3566 \%$ |
| $2.99 €$ | $0.3241 \%$ |
| $6.99 €$ | $0.2924 \%$ |
| $3.99 €$ | $0.2914 \%$ |
| $1.49 €$ | $0.2869 \%$ |
| $3.00 €$ | $0.2720 \%$ |

Fig. 2: Frequent basket sums

Obviously, different basket sums are not equally distributed. Some payment sums are much more frequent than they would be in case of an equal distribution.

Basket sums are not equally distributed

Information on payment sums

Relations between distributions of basket sums, payment sums and change amounts

A formulised answer
$f$ operates on the whole distribution

Consequentially, other basket sums are much less frequent. To illustrate this finding Figure 2 shows the ten most frequent payment sums.

All in all 16.319 different basket sums were observed. For this number of different sums the average relative frequency would only be around $0.0061 \%$. As Figure 2 shows, some basket sums have come up more than 80 times more frequently than one would have expected in case of an equal distribution. Another result of Figure 2 is the fact that the nine most frequent basket sums all end on the digit 9 . All but one of these even end with 99 .

The database of market baskets along with basket sums contains payment sums as well. Since a simple subtraction of these numbers gives the change amount, a distribution of change amounts can also be calculated from the data. Before we show this empirical distribution some additional considerations seem to be in order.

The distribution of change amounts is determined by the distribution of payment sums. This, in turn, depends on the distribution of basket sums since different distributions of basket sums would lead to different distributions of payment sums. One question which immediately comes up in connection with these three distributions is the following: How exactly is the relation between the distribution of basket sums and the distribution of change amounts?

This question asks for a function $f$ with $f\left(V_{\mathrm{B}}\right)=V_{\mathrm{C}}$, where $V_{\mathrm{B}}$ is the distribution of basket sums and $V_{\mathrm{C}}$ is the distribution of change amounts. The two distributions can be written as vectors of tuples with two components each: ( $\left\langle x_{i} ; y_{i}\right\rangle,\left\langle x_{i+1}\right.$; $y_{i+1}>\ldots$ ). In each tuple, a relative frequency $y_{\mathrm{i}}$ is assigned to a specific sum $x_{\mathrm{i}}$. For the data of Table 1 such a vector would look like this: (<5.99; 0.4869>, <0.99; $0.4544>$...).

There are two basic ways in which the function $f$ could operate.

1. The function $f$ could operate on the distribution of basket sums as a whole. For example, it could work in the following way: For the empirical distribution $V_{B}$ the parameters of a specific theoretical distribution model are estimated. The estimated values of these parameters now determine the values of the parameters of another theoretical distribution model. The distribution described by this second model with the given parameters is taken as the result of $f$ and thus as an estimate for $V_{\mathrm{C}}$.
2. But $f$ could also operate on the precise basket sums one after another. For each basket sum probabilities for different possible payment sums have to be calculated. From these, the probabilities of different change amounts are inferred. Using the relative frequencies of the different basket sums the results for all basket are now integrated, yielding a new distribution that could be used as an estimate for $V_{\mathrm{C}}$.

The first kind of function, working on aggregated data, does not seem to be adequate to the problem, because the precise basket sums as such do only have an intermediate role. In particular, a function $f$ operating in this way could not really be interpreted as a hypothesis for the payment behavior of consumers. Rather it would be a hypothesis on the results of the unknown payment behavior of a group of people. For this reason we follow the second operating principle for the function $f$ sketched above.

Here, the operation of the function $f$ can be defined in the following way: A specific function $g$ is processed with each tuple of $V_{\mathrm{B}}$ one after another as inputs. The result of each execution of $g$ is a new distribution that can be written as a set of tuples with two components each as explained above. After $g$ has been used with every tuple in $V_{\mathrm{B}}$, all the vectors resulting from these operations are combined to form the overall result. This combination is achieved by adding the respective second components of tuples in which the first components are equal. We give a simple example to illustrate the process: Let $V_{\mathrm{B}}$ be ( $<1 ; 0.25>,<2$; $0.75>$ ). To simplify the calculation we further assume that $g$ is the same for all basket amounts in $V_{\mathrm{B}}$ and gives a probability of $50 \%$ each for a precise payment and a payment of double the required amount, respectively. With this we get: $g(<1 ; 0.25>)=(<1-1 ; 0.5 \cdot 0.25>,<2-1 ; 0.5 \cdot 0.25>)$ und $g(<2 ; 0.75>)=(<2-$ 2; $0.5 \cdot 0.75>,<4-2 ; 0.5 \cdot 0.75>$ ).
The subtractions in the first components of the resulting tuples must take place because we want to infer change amounts from basket sums and payment sums which is done by subtracting the basket sum from the payment sum.
Combining the two new vectors yields (<0; 0.5>, <1; 0.125>, <2; 0.375>). This result contains only three tuples since one tuple could be discarded because two tuples had the same first component (with the value 0 ).

Since $V_{B}$ and $V_{C}$ are known from the market basket database, the specific definition of $f$ would be the answer to the question concerning the relation between the distributions of basket sums and change amounts. The general
$f$ operates on the basket sums

Aggregate function does not catch behavior but impact of behavior

## Definition of

 disaggregate $f$Possible criteria in consumers' payment decisions

Number of tokens
mechanic of $f$ has already been defined. Thus, only an appropriate $g$ is lacking to answer this question: We look for a function $g(x, y)$ for all tuples $\left\langle x ; y\right.$ in $V_{\mathrm{B}}$.

Some considerations must be taken into account with respect to such a function $g$.
Not every customer always has the coins and banknotes needed to compose a specific payment sum. If his money allows to compose various payment sums bigger than the basket sum it is unknown which one he will choose. Specifically, it cannot be assumed that he will give exactly the required amount even in situations in which this is possible.
Beyond the restrictions given by the coins and banknotes available in customers' purses, quite a few other criteria could apply to their decisions concerning payment sums:

1. The number of used coins or banknotes: 1a. For the payment sum: Customers pick the payment sum requiring the smallest number of coins and banknotes to compose. 1b. For the change amount: Customers pick the payment sum that, assuming optimal composition of the change amount, i. e. composition with the smallest possible number of coins and banknotes, yields the smallest number of coins or banknotes in the change amount. 1c. For payment sum and change amount combined: Customers pick the payment sum that, assuming optimal composition of the change amount, minimizes the number of coins and banknotes that have to be 'moved' for both sums.

Such a criterion could explain why customers do not compose the exact amount required even when they are able to do so: From the perspectives of some customers the effort to compose the exact amount could entail too much effort. For example, the composition of exactly 1.48 Euro requires at least six coins (100, 20, 20, 5, 2, 1) whereas payment sums of 1.50 Euro or 2 Euro each only lead to three coins for payment sum and change amount combined. Even the retailer is not guaranteed to be very happy with all payment sums being composed exactly: Counting and ordering all the coins could very well take more time than composing and handing out change amounts.
2. The weight of the coins and banknotes used: 1a. Minimizing the amount of 'moved' currency: Customers pick the payment sum that, assuming optimal composition of change money with respect to its weight, minimizes the weight of payment sum and change amount combined. Since customers do not know whether the cashier will weight-minimize the change amount (especially because weight-minimizing in general yields another configuration of the
change amount than number-minimizing) it could be an alternative for customers to directly minimize the weight of the payment sum. 2a. Minimizing the weight of coins and banknotes customers hold after the transaction: Customers pick the payment sum that, assuming weight-minimizing of the change amount, minimizes the weight of coins and banknotes left in customers' purses after transactions. Since it is unknown for customers whether the cashier will weight-minimize the change amount it could be an alternative for customers to directly maximize the weight of the payment sum.
3. The type of coins and banknotes handed out for change: Some customers might pick their payment sums with the types of coins or banknotes in view that they would presumably get as change, e. g. when assuming numberminimizing composition of change amounts by the cashier. For example, a customer could react with a payment of 24 cent instead of 20 cent to a basket sum of 14 cent if he wants to get back a 10 -cent-coin.

Such a wish could be determined by different motives. Customers may need a specific coin or banknote for a specific later purpose, e. g. for feeding a vending machine. But such a motive could also be a subtype of the first criterion: In the given example the customer would minimize the number of coins he has to 'carry away': Instead of two coins in case of a payment sum of 20 cent ( 5 cent, 1 cent) he only gets back one coin ( 10 cent) and could also get rid of more coins since composing 24 cent requires more coins than composing 20 cent. All in all this customer has minimized the number of coins available to him after the transaction.

Let us assume that such criteria and other similar ones were in fact important for customers' payment decisions. In addition to that, there can be intrapersonal and interpersonal differences the payment behavior of different customers.
An intrapersonal difference would exist if one and the same customer uses different criteria to decide on the payment sum when reacting to different basket sums. But even in reaction to one and the same amount at different occasions he might use different criteria. This could be explained, of course, by different sets of available coins and banknotes at each point in time. But also circumstances which are not directly connected to the transaction could motivate such a difference, e. g. the fact that the customer sometimes wants to use a vending machine directly after the transaction and thus requires specific coins.

An interpersonal difference is given if two customers do not react in the same way to the same basket sum although both have the same set of coins and banknotes

Type of tokens

Intrapersonal variations of payment behavior

Interpersonal variations of payment behavior
$g$ yields vectors of different lengths

Specific results are less important than method

Reasonable payment sums
available. This could be explained by different preferences of the customers: one might be bothered more by a heavy, the other more by a voluminous purse.

Because of these differences empirical distributions of change amounts cannot be explained by assigning one specific payment sum to each basket sum. I. e., the unknown function $g(x, y)$ has to yield vectors with differing numbers of tuples depending on the value of $x$. The types of these distributions represent the payment behavior of the customers who bought the analyzed market baskets. For example, if all customers would always give a payment sum equalling the basket sum the following equation would hold: $g(x, y)=(<0 ; y>)$.

The aim of this paper is foremost conceptual. Thus, it is primarily concerned with the way to do payment analysis and only after that with the specific results obtained by analyzing the given market basket data. Because of that we have only put a limited amount of effort into finding a good function $g$, i. e. a $g$ such that the $c$ in $f\left(V_{\mathrm{B}}\right)-V_{\mathrm{C}^{*}}-c=0$, which is the difference between the empirical distribution of change amounts $V_{\mathrm{C}^{*}}$ and the distribution of change amounts estimated by $f$, is minimal. (To measure the difference between $f\left(V_{\mathrm{B}}\right)$ and $V_{\mathrm{C}^{*}}$ we use the average relative differences of the two distributions for all change amounts.)

To further explain our function $g$ we now use the symbols from section 2 to define the notion of a reasonable payment sum: A payment sum $N$ with $N=\sum_{i}$ 1SiSD $m_{i} n_{i}$ is a reasonable payment sum given the basket sum $B$ if the following inequality holds for all $i: N-n_{\mathrm{i}}<B$, i. e. no coin and no banknote can be subtracted from $N$ without decreasing $N$ to a sum smaller than $B$. Example: A basket sum of 1.48 Euro is given. With the given euro coin system there are various reasonable payment sums: 1.48 Euro, 1.49 Euro, 1.50 Euro, 1.60 Euro and 2.00 Euro. With these payments sums change amounts of 0.00 Euro ( 0 coins), 0.01 Euro ( 1 coin), 0.02 Euro ( 1 coin), 0.12 Euro ( 2 coins) or 0.52 Euro ( 2 coins) would be associated. It is easily seen why other payment sums are not said to be reasonable: E. g., each possibility to compose a payment sum of 1.70 Euro would allow the cashier to return at least one coin to the customer without decreasing the sum under 1.48 Euro. Since this is the case, this coin can be ignored when the number of coins needed to compose the change amount is calculated.

Empirical results on the use of reasonable payment sums

An analysis of all the market baskets with a basket sum under 15 Euro from the database described above shows that in around $16 \%$ of transactions buyers gave a payment sum that is unreasonable according to the given definition. For all
baskets with a basket sum under 200 Euro the fraction of unreasonable payment sums rises to $21.3 \%$. The fact that the fraction of unreasonable payment sums seems to increase with the basket sums could be explained by assuming that buyers with bigger basket sums increasingly try to compose payment sums that allow change amounts entirely composed by banknotes. To achieve this, in many cases unreasonable payment sums have to be used. For example, when composing 54.50 Euro to meet a basket sum of 44.50 Euro and enable a change amount of 10.00 Euro, the buyer needs at least three coins which could be directly returned to him without depriving his payment sum of the capability to meet the basket sum.

Since most of the analyzed transactions showed reasonable payment sums our function $g$ is based on the concept of reasonable payment sums. The first step in the operation of this $g$ is calculating all reasonable payment sums for a given basket sum.

Relatively small change amounts are very frequent. Thus they get a special treatment. To enable this the determined reasonable payment sums are divided into a group of fitting and a group of unfitting payment sums.

Treatment of fitting payment sums: The probability $p(b, z)$ for a fitting payment $z$ in reaction to a specific payment sum $b$ can be approximated very well as a function of the number of coins or banknotes required to number-minimize the payment sum $\min (b): p(b, b)=(\ln (\min (b)) \cdot-0.2359)+0.4772$. Figure 3 shows the combinations of the 2000 most frequent basket sums with their respective relative frequency of a fitting payment as black dots. The gray curve plots the given function $p$.

The negative relation between the number of coins needed to compose a payment sum equalling the basket sum and the probability of such a composition actually taking place is easily understood: The more coins and banknotes a buyer would need to compose a given sum the less he bothers to. E. g., in cases with basket sums of 15.00 Euro fitting payments are more probable than in cases with payment sums of 17.37 Euro.

Fitting payment sums


Fig. 3: Number of composition coins and probability for fitting payment sums

Thus, for each tuple $\left\langle x_{\mathrm{i}} ; \mathrm{y}_{\mathrm{i}}\right\rangle$ of $V_{\mathrm{B}}$ the function $g$ first calculates a tuple $\left\langle x_{\mathrm{i}}-x_{\mathrm{i}} ; y_{\mathrm{i}}\right.$ $\cdot \mathrm{p}\left(x_{\mathrm{i}}, x_{\mathrm{i}}\right)>=<0 ; y_{\mathrm{i}} \cdot \mathrm{p}\left(x_{\mathrm{i}}, x_{\mathrm{i}}\right)>$, i. e. the probability for a fitting payment equalling a change amount of zero is calculated by multiplying the relative frequency $y_{i}$ of the basket sum $x_{\mathrm{i}}$ with the probability that $p\left(x_{\mathrm{i}}, x_{\mathrm{i}}\right)$ that this basket sum is met by a fitting payment sum.

Treatment of unfitting payment sums: After treating the fitting payment sums a relative frequency amounting to $\left(y_{\mathrm{i}} \cdot\left(1-\mathrm{p}\left(x_{\mathrm{i}}, x_{\mathrm{i}}\right)\right)\right)$ is left to be distributed onto the unfitting reasonable payment sums (and resulting change amounts) for the given basket sum.

The empirical investigation has shown that payment sums for given basket sums get less probable while increasing. In a process of trial and error we designed a function expressing this relation. After that, this function was modified step by step to decrease $c$ in $f\left(V_{\mathrm{B}}\right)-V_{\mathrm{W}^{*}}-c=0$. We abstain from showing the best function $g$ reached in this way here because it is rather complex and probably not even near the best possible solution (note that we did not intend to find an optimum but to demonstrate how one could look for one).

We now used this function $g$ to calculate values for the different unfitting payment sums. The relation of these values was then used as the key to distribute the remaining part of the basket sum's relative frequency between these payment sums. So $g$ was completed for unfitting payment sums as well and could be used to calculate the estimate $f\left(V_{\mathrm{B}}\right)$ for the distribution of change amounts.

Figure 4 shows the distribution $V_{\mathrm{C}^{*}}$ as a black and its estimation $f\left(V_{\mathrm{B}}\right)$ as a grey curve in a cumulative chart. It is visible that the estimation deviates from the empirical observations. For change amounts under 10 Euro and thus for more then $70 \%$ of all transactions the estimation is quite good whereas for other ranges of change amounts the difference between reality and estimation is substantial. A further optimization of $g$, specifically with regard to the treatment of unfitting payment sums, appears to be possible, increased calculation power given. Basically, the problem is to find a function $g$ encoding the probabilities of various reactions to given basket sums such that combining the results for all single basket sums yields a distribution of change amounts roughly equal to the one observed in the data.


Fig. 4: Observed and estimated distributions of change amounts

## 5. Managerial implications

High speed and low costs of cash desk operations conflicting aims

Practical use of payment analysis

Using payment analysis for Rationalization

Cash desks indicating the types of coins to use

Change composition in order to make change money last longer

Under normal circumstances retailers can be assumed wanting to have cash desk processes as fast and as cheap as possible. Unfortunately there is no clear relation between these two goals: Payments could in fact be made faster by using applicable chip cards, but it is unclear, whether money saved this way could cover additional costs for hardware and services necessary to enable payment by chip cards. In the market basket database used in this research only approximately $7 \%$ of all payments were not done in cash, i. e. with credit or debit cards, so that the company from which our market basket database originates could apply the results of a payment analysis to most of his transactions. It should be noticed that payment with these types of card needs more time than with other techniquely possible chip cards, which could be used just like special banknotes which always have the right denomination, because either a PIN has to be entered by the payer or a document has to be signed. In addition to that often a data connection to the computer of a bank or a credit card provider is necessary which uses time and money as well. The question, whether retailers should enable non-cash payment and which specific types of this should be allowed, cannot be further addressed here.

How could a retailer take advantage of the results of payment analysis? Different directions of using this kind of information shall be discussed shortly.

On the one side there are ways to use results from payment analysis to make processes at the cash desk more efficient.
The behavior of cash desk personnel could be influenced with these results in view: For example, the cash desk itself in addition to the change amount could also show which coins the cashier should use to compose the amount due, possibly by using LED-Displays at the different compartments of the cash box. Doing this could make the composition of change faster and the occurrence of errors more unlikely.
The cash desk could be made to not only know the amount of cash that is supposed to be stored in it at a given moment in time but also the number of each different denominations present in it. This could be realized using a cash box with weights for each compartment. If the cash desk knew which denominations abound and which are scarce it would be able to direct the composition of change money so that more of the abundant denominations are used. The way change is composed would in this way be made directly dependent on the way payers
compose their payment sums. This would enable cash desks to operate longer without supply of new coins for making change. Costs that result from interest losses because of cash holdings and also from using security services to transport cash between banks and outlets could be reduced by such a measure.
As long as a significant fraction of all transactions involve payment in cash retailing will have to deal with the problems of cash management. A new idea shows how costs in connection with this could be further reduced: Using special cash desks and in cooperation with banks retailers in the USA begin to avoid interest losses by putting money into accounts bearing interest as soon as it enters the cash desks.
Another step further not only involves minimizing the amount of change money but also giving back cash that has come in directly to payers who need it: Again in cooperation with banks retailers could operate automated teller machines just like banks. Ideally, a customer $B$ would be provided by the retailer with cash that customer $A$ just spent at a cash desk. The bigger the number of customers is who only supply themselves with cash after entering the outlet, e. g. because this helps them avoid the need to visit their local bank, the smaller the amount of cash is that retailer have to handle themselves. This kind of operation could transform cash from a cash payment directly into money arriving on retailers' accounts because, for example, it comes from a customer's account with a credit card company. Retailers could even try to save money through enabling fewer different payment methods at cash desks as long as customers are enabled to supply themselves through the debit or credit card of their choosing directly inside the outlet.
Technically it seems to be possible to integrate such a function of obtaining money from telling machines in existing or new customer card programs which would lead to a further enrichment of the data obtained through using these programs and with that to more possibilities of data and customer analysis.

The results of payment analysis not only can be used to reduce the amount of change money needed. The empirically measured distribution of change amounts in a retail outlet can also be used to forecast the amount of change money needed in the future. This would help to reduce the costs of cash management in the outlet. By comparing the results of different outlets it would moreover be possible to detect similarities and to decide if a centralization of cash management to an organizational level above single outlets would be promising.

Apart from these foremost technical ways of process optimization the results from payment analysis could also be used for new kinds of promotions.

Cash desks carrying interest

ATM could minimize cash transports

Forecast of change money needed in the future

Rebates related to basket sums

Simplify change making while retaining advantages of fraction prices

Starting from a specific basket sum scaled percentage discounts could be given, for example $1 \%$ starting from basket sums of 20.00 Euro and $2 \%$ from 50.00 Euro: Payment analysis can deliver a projection of the costs of such a promotional activity which allows an informed decision on the feasibility of such a measure. But one can also think about less classical kinds of promotions: For example, if payment analysis shows significantly more basket sums between 45.00 and 50.00 Euro than between 50.00 and 55.00 Euro, a promotion of the kind "Each $x$-th customer with a basket sum of exactly 50.00 Euro gets a bonus $y$ " would be promising because in this case probably more customers from the 45.00-50.00 range would buy more to get the bonus than customers from the 50.00-55.00 range would by less to get it. All in all this could result in more sales. Since the proportions of these groups are known through payment analysis a good projection of the outcome of such a promotion could be made, enabling managers to make a better decision.

In Finland there are only very few 1- and 2-Cent coins in circulation because as a result of specific laws almost all prices end on the digits 0 or 5 . The introduction of these measures is rooted in a Finnish tradition because the small denominations of the old currency Finnmark were also widely out of use (similar discussions are currently going on in other countries, e.g. in the Netherlands and Austria). Having the concept of prices ending on the digits 7 or 9 in view, further research into the question whether consumer behavior in Finland differs from consumer behavior in Germany or elsewhere would be interesting. If this was found not to be the case the Finnish example could point the way to a more simple pricing policy for retailers which would entail easier composition of change amounts since change amounts of 5,10 or 15 Cent are much easier to compose than the amounts between them. In Germany for each food product the price per kilogram has to be visible for customers even in cases where the actual weight of it is only 0.1 or 0.2 kg . Depending on the actual sizes of the product the kilogram prices could still end in the digits 7 or 9 or just below suspected psychological barriers even if the actual prices of the products ended with the digits 0 or 5 . Using a prominent placement of the kilogram price could thus enable retailers to reap the benefits of simple prices and simple change making without losing the apparently widely believed in effects of prices just below full amounts.

## 6. Summary

The present paper has dealt with a topic that has found a surprisingly small amount of interest in the field of marketing and business research: cash. Money as a unit of calculation of course is prominent enough but for the physical side of money, the handling of coins, this is not the case. There are good reasons for this fact: The organiszation of the actual money circulation is rather a technical question, concerned with metals, denominations, coin weights or, in recent times, electronic payment systems.

But there are also interesting questions for business researchers in connection with the physical side of money and we have tried to highlight a few of them above.

The empirical basis of the research has been a market basket database containing more than 1.5 million market baskets from the everyday operational business of a middle sized German retailing company. This database has been used to calculate the frequencies of specific basket sums and change amounts found in reality. After discussing a way to express hypotheses on the relation between the two in a functional form we compared these empirical results those predicted by a sample hypothesis on consumers' paying behavior. The fit was not perfect but appears close enough to make this hypothesis or similar ones useful for retailers.

It is not easy to see how far our empirical results and inferences drawn from these can be transferred to data from other retailers. Some results can be expected to be representative for similar companies, for example the frequencies of different basket sums or change amounts. An even more general result is the fact that in approximately $21 \%$ of all cases payment sums are composed which are not rational. This finding points to the fact that customers anticipate specific change compositions. On the other hand the fraction of only $7 \%$ of transactions that involve payment with other means than cash seems to be rather small.

The hints given in section 5 concerning possibilities for retailers to take advantage of the results of payment analysis can only be regarded as a first attempt. Further research could involve case studies specifically exploring the possible benefits of rationalization or promotions done on the basis of payment analysis.

All in all it seems to be a convincing statement that retailers can gain access to possible improvements of processes and decisions through payment analysis which outweigh the costs of this type of analysis.

Can the results be generalized?

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