Energy generation with Directed Technical Change*

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Abstract

To analyze the direction of technical change with respect to energy generation, we use a growth model with two cost reducing technologies, one associated with an exhaustible resource and one with a backstop. We show that research in the exhaustible resource sector is not optimal in the long run. Furthermore, it can be only conducted in the short run, if the economy is sufficiently rich. Otherwise, the temporally limited effect of R&D in the resource sector is outweighed by the unlimited effect of R&D in the backstop sector or of capital accumulation. If the exhaustible resource causes pollution, the resource is partly substituted by the backstop. Therefore, the attractiveness of research in the resource sector is reduced while research in the backstop sector gets more attractive. We not only describe the direction of technical change but the complete economic development. With two non-exhaustible energy sources, R&D is conducted in both sectors in the long run. Thus, the limited resource stock alters the direction of technical change fundamentally.

Key words: Fossil Fuels, Energy, Renewable Resources, Endogenous Growth, Directed Technical Change

1 Introduction

At least since the 1970s scarce fossil fuels are one of the major issues both in public and academic discussion. While the famous Club of Rome report (Meadows et al. (1972)) predicts an end of all economic growth due to exhaustible resources, a broad strand of economic literature advocates a more optimistic viewpoint. E.g. Stiglitz (1974) shows in a neoclassical growth model that sustainable growth is possible as long as technology, which serves as a substitute for an exhaustible resource, grows

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fast enough.[1] Barbier (1999) endogenizes the technological progress without altering the result substantially.[2] The possibility of sustainable growth in consideration of a backstop technology, such as solar energy or fusion power, is shown by Tsur and Zemel (2005). The development of alternative energy sources is addressed by Tsur and Zemel (2011).
However, it was not until the works of Acemoglu (1998), Kiley (1999) and Acemoglu (2002) that the effect of scarce resources on the direction of technical change have been analyzed. More precisely, Acemoglu (2002) assumes that a composite good is produced by means of two intermediate goods using a CES production function. The production of both intermediates is based on inputs like capital or labor and a particular technology. Both technologies can be improved by R&D. Di Maria and Valente (2008), Grimaud and Rouge (2008) and Acemoglu et al. (2012) have augmented this framework with exhaustible resources. They assume that an exhaustible resource, like fossil fuels, is needed for the production of one of the two intermediates. It is shown that the direction of technology progress depends crucially on the substitutability of the resource based intermediate and the second intermediate. If the intermediates are good substitutes, i.e. the resource based intermediate is not necessary for production, research is only conducted in the sector of the non-resource based intermediate in the long run, as demonstrated by Acemoglu et al. (2012). On the other hand, Di Maria and Valente (2008) show that R&D efforts are completely allocated to the sector of the resource based intermediate, if the intermediates are bad substitutes.

However, two aspects of the literature seem critical if energy generation is taken into account explicitly. Firstly, renewable energy sources like solar or wind energy are ignored, albeit they are already used and may serve as backstops after the exhaustion of fossil fuels. The second critical aspect is the resource augmenting technology. This assumption is common in the endogenous growth theory and crucial for the optimistic results of Stiglitz (1974), Barbier (1999) and Di Maria and Valente (2008). In the long run, the assumption allows the production of a vast amount of energy by means of a vanishing low amount of resources and a sufficiently advanced technology. With respect to fossil fuels this property can be hardly brought into accordance with thermodynamics, which require a minimum stake of resources.[3]

The intention of our paper is to analyze the effects of energy generation based

[1]Stiglitz (1974) belongs to a series of articles which can be seen as a direct response to Meadows et al. (1972). Other works are Dasgupta and Heal (1974), Solow (1974) and Hartwick (1977).

[2]Barbier (1999) stands in the tradition of the endogenous growth theory. An overview of this broad strand of literature is given by Aghion et al. (1998) as well as by Barro and Sala-i Martin (2003).

[3]Compare Meyer et al. (1998), page 171.

on both fossil fuels and backstops on economic development and especially on the direction of technical change without these critical assumptions. For this purpose we augment the endogenous growth model of Tsur and Zemel (2005) with a second technology. Green and black energy are perfect substitutes in this model. Furthermore, R&D reduces the costs of energy supply. We characterize the complete optimal development process, which encompasses the optimal capital investment, R&D investments both in the green and the black energy sector (black and green research), consumption and the energy mix at every point in time. The capital endowment and the ability to conduct research in the two energy sectors affect the process, which is driven by a competition between consumption and investments in either one of the research sectors or the capital stock over shares of an universally usable final good. Since the two energy sources are perfect substitutes and the technical progress not resource augmenting, the attractiveness of black research vanishes with the resource stock. Therefore, it is not optimal to conduct black research in the long run. In the short run, black research can be only optimal for an economy with a sufficiently high capital stock, i.e. a rich economy. In contrast, green research can be optimal for rich as well as poor economies in both the long and the short run. If the exhaustible resource pollutes the environment, e.g. by generating CO_2 emissions, her optimal utilization rate is lower than without pollution. This affects negatively the attractiveness of black research but boosts the one of green research, as black energy is substituted by green energy. In a decentralized economy, the researching companies exert market power, i.e. the utilization rate of both green and black energy are inefficiently low. A subsidy for both energy sources can internalize the market power distortion. If the exhaustible resource causes pollution, a tax on the resource is sufficient to internalize environmental pollution.

Our model differs from Acemoglu et al. (2012) with respect to the research approach, too. The "standing on the shoulders of giants" R&D approach, used by Acemoglu et al. (2012), drives a wedge between the productivity of R&D in the two research fields. If both energy sources are non-exhaustible, this effect shifts R&D to the more advanced sector. In contrast, we apply the lap equipment approach, which eliminates such a wedge. Therefore, R&D investments are split among both research sectors in the long run. [4]

Furthermore, we determine not only the direction of technological change but also the optimal levels of R&D, consumption and capital accumulation. Thus, in contrast to Acemoglu et al. (2012) and Di Maria and Valente (2008) R&D investments

[4] Both research approaches are analyzed by Rivera-Batiz and Romer (1991) in an endogenous growth model.

can be abandon in favor of capital accumulation or a steady state.

The outline of the paper is as follows. In section 2 we describe the model and derive the conditions characterizing the optimal solution. Section 3 describes the optimal development process with two non-exhaustible energy resources, while the exhaustibility of one resource is taken into account in section 4. We augment the model with pollution in section 5 and analyze governmental interventions necessary for the realization of the social optimum in a decentralized economy in section 6. Section 7 concludes the paper.

2 Model

We augment the endogenous growth model of Tsur and Zemel (2005) with a second technology. The basic framework of the augmented model is described in the following.[5] A single composite good Y is produced by means of capital K and energy x. The production function $F(K,x)$ has positive but diminishing marginal products F_K and F_x, i.e. $F_{KK} < 0$, $F_{xx} < 0$. Furthermore, $F(0,x) = F(K,0) = 0$, $F_{Kx} = F_{xK} > 0$ and $J = F_{KK}F_{xx} - F_{Kx}^2 > 0$, implying the concavity of $F(K,x)$. Energy is generated by means of an exhaustible resource R (coal, oil, gas) and a backstop resource b (solar, fusion power), respectively, according to proportionate transformations. We assume $x = R + \nu b$, where the constant marginal rate of substitution $\nu \in]0,1]$ reflects infrastructure adjustments associated with changes in backstop utilization. We think here of a more complex high voltage network, energy storage solutions necessary for reliable energy supply based on renewables and conversion losses, e.g. the energy input needed for the synthesis of liquid fuels.[6] All mentioned issues imply energy losses, explaining $0 < \nu < 1$. In the case of fusion power it is reasonable to assume that the existing infrastructure can be used with only minor adaptions implying a high ν.

The supply of both resources causes costs. The extraction costs of the exhaustible resource are given by $M(R)B_R(A_R)$. $B_R(A_R) > 0$ represents the effect of the (black) technology A_R on the extraction costs, while the "pure" or technology adjusted extraction costs are $M(R)$. We assume that the extraction costs as well as the marginal extraction costs are increasing in the extracted amount R, i.e. $M'(R) > 0$ and $M''(R) > 0$. Additionally, we assume $M(0) = M'(0) = 0$. Technological progress reduces the extraction costs, i.e. $B_R'(A_R) < 0$. However, this effect diminishes as technology increases and vanishes completely for $A_R \to \infty$. Thus, $B_R''(A_R) > 0$ and

[5] For the sake of simplicity time index t is suppressed. It is only added, if needed for understanding.

[6] The first two points are major issues in Germany currently, due to its energy turnaround.

$\lim_{A_R\to\infty} B'_R(A_R) = 0$. We assume $\lim_{A_R\to\infty} B_R(A_R) = \bar{B}_R > 0$, so that technology can not lower the extraction costs of a marginal resource unit below a specific threshold $\bar{B}_R$. The costs of the backstop are $M_b B_b(A_b)b$, with $M_b b$ as the "pure" or technology adjusted costs, which are linear in backstop utilization. Similar to $B_R(A_R)$ the function $B_b(A_b) > 0$ represents the effect of the (green) technology A_b on backstop costs. $B_b(A_b)$ exhibit the same properties as $B_R(A_R)$, i.e. $B'_b(A_b) < 0$, $B''_b(A_b) > 0$, $\lim_{A_b\to\infty} B_b(A_b) = \bar{B}_b > 0$ and $\lim_{A_b\to\infty} B'_b(A_b) = 0$. The costs for resource utilization are deducted from production, so that the net production is given by

$$Y^n = F(K,x) - M(R)B_R(A_R) - M_b B_b(A_b)b. \tag{1}$$

Both technologies increase linear in R&D investments $I_i,\ i = R, b$, i.e.

$$\dot{A}_i = I_i,\ i = R, b. \tag{2}$$

R&D investments as well as capital investments and consumption are using the composite good Y. Therefore, the net production Y^n can be either consumed or invested in capital and research, respectively. With C as consumption, the (dis)investments in capital $\dot{K}$ are

$$\dot{K} = F(K, R + \nu b) - C - M(R)B_R(A_R) - M_b B_b(A_b)b - I_R - I_b. \tag{3}$$

The sum of I_b and I_R is limited by net production such that $I = I_b + I_R \in [0, \bar{I}]$, with $\bar{I} = Y^n$. The stock S_R of the exhaustible resource, with the initial value S_{R_0}, decreases according to

$$\dot{S}_R = -R. \tag{4}$$

The representative household derives its utility only from consumption. The utility is given by the increasing and strictly concave function $U(C)$, i.e. $U'(C) > 0, U''(C) < 0$.

In the following section we analyze the social optimum. A social planer maximizes the present value of utility $\int_0^\infty U(C)e^{-\rho t}dt$, with ρ as the time preference rate, over the complete planing period $[0,\infty[$, given the initial states $(K_0, A_{b_0}, A_{R_0}, S_{R_0})$ and subject to (3), (2), (4), $K \geq 0$, $S_R \geq 0$, $0 \leq I_R + I_b \leq \bar{I}$ and $R, b, C \in [0,\infty[$. With λ, κ_R, κ_b and τ representing the current-value costate variables of K, A_R, A_b and S_R and the $\zeta_i, i = K, S_R, \bar{I}, I_b, I_R, R, b, C$ denoting the Kuhn-Tucker multipliers, the current value Lagrangian is

$$\begin{aligned} L = U(C) &+ \lambda[F(K, R + \nu b) - C - M(R)B_R(A_R) - M_b B(A)b - I_R - I_b] \\ &+\kappa_R I_R + \kappa_b I_b - \tau R + \zeta_K K + \zeta_{S_R} S_R + \zeta_{\bar{I}}[\bar{I} - I_b - I_R] + \zeta_{I_b} I_b \\ &+\zeta_{I_R} I_R + \zeta_R R + \zeta_b b + \zeta_C C. \end{aligned} \tag{5}$$

An interior optimum with respect to capital, resource stock, resource and backstop utilization and consumption, i.e. $\zeta_K = \zeta_{S_R} = \zeta_R = \zeta_b = \zeta_C = 0$, is given by the necessary conditions[7]

$$U_C = \lambda, \tag{6}$$

$$F_x = M'(R)B_R(A_R) + \frac{\tau}{\lambda}, \tag{7}$$

$$F_x = \frac{M_b}{\nu} B_b(A_b), \tag{8}$$

$$\lambda = \kappa_b - \zeta_{\bar{I}} + \zeta_{I_b} = \kappa_R - \zeta_{\bar{I}} + \zeta_{I_R}. \tag{9}$$

(7) and (8) give

$$F_x(K, x(K, A_b, A_R)) = M'(R(K, A_b, A_R))B_R(A_R) + \frac{\tau}{\lambda} = \frac{M_b}{\nu} B_b(A_b). \tag{10}$$

Equation (10) determines the total amount of energy utilization x as well as the shares of the two energy resources b and R as illustrated in Fig. 1.[8]If the marginal costs of the backstop are to high, only the exhaustible resource is used. The total energy equals then the amount $R^{\#}$. If $M'(0)B_R(A_R) + \frac{\tau}{\lambda} < \frac{M_b}{\nu} B_b(A_b) < M'(R^{\#}) + \frac{\tau}{\lambda}$ holds for the marginal backstop costs, energy generation is based on both resources. The backstop is used alone, if $\frac{M_b}{\nu} B_b(A_b) < M'(0)B_R(A_R) + \frac{\tau}{\lambda}$. Following Tsur and Zemel (2005), we assume the simultaneous use of both resources. This implies that the amount of total energy supply is independent from A_R as can be seen in Fig. 1. The term $\frac{\tau}{\lambda}$ of the exhaustible resource supply function relates the shadow price of the exhaustible resource to that of capital. Since the shadow prices of a variable indicate its scarcity, the term measures the relative scarcity of the

[7] In the sequel we assume that the sufficient conditions are satisfied. They hold as long as $M(R)B''_R(A_R)B''_b(A_b) \geq \frac{M'(R)^2}{M''(R)} \frac{B'_R(A_R)^2}{B_R(A_R)} \left[\frac{F_{KK}}{J} \frac{M_b}{\nu^2} \frac{B'_b(A_b)^2}{b} + B''_b(A_b)\right] + M(R)\frac{M_b}{\nu^2} \frac{B'_b(A_b)^2}{b} B''_R(A_R) \left[\frac{1}{M''(R)B_R(A_R)} - \frac{F_{KK}}{J}\right]$, $M(R)B''_R(A_R) \geq \frac{M'(R)^2}{M''(R)} \frac{B'_R(A_R)^2}{B_R(A_R)}$ and $B''_b(A_b) \geq \frac{M_b}{\nu^2} \frac{B'_b(A_b)^2}{b} \left[\frac{1}{M''(R)B_R(A_R)} - \frac{F_{KK}}{J}\right]$. It can be shown that the last two conditions hold for $b > 0$ and $R > 0$ if M_b is sufficiently small and $M(R)$ sufficiently high, respectively. If both conditions hold with inequality, the first conditions will hold, too.

[8] Tsur and Zemel (2005) use a similar figure without the second technology A_R.

exhaustible resource in terms of capital scarcity. Therefore, we call it the relative scarcity index.

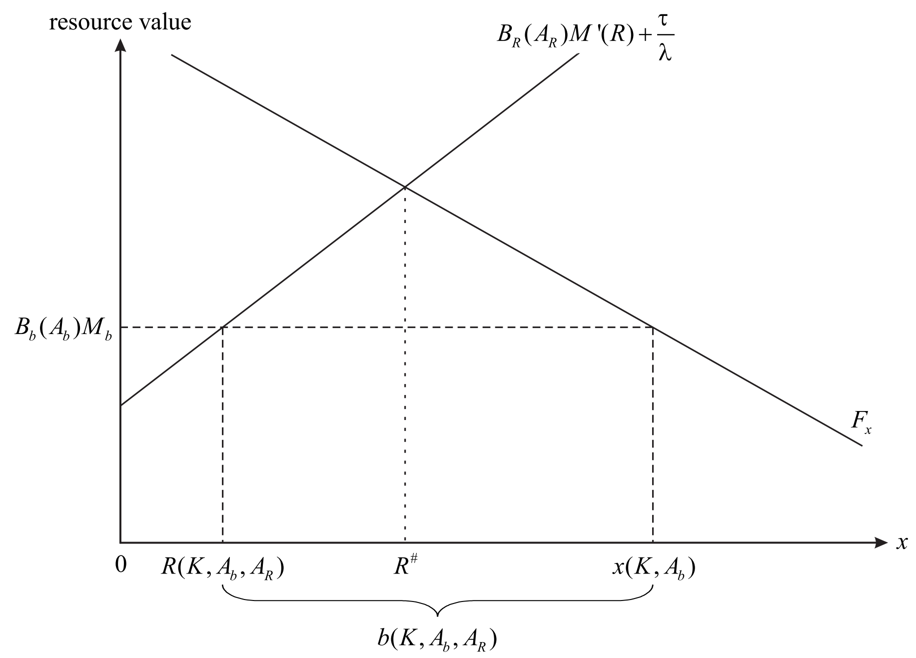

Figure 1: Utilization of exhaustible resource and backstop

In the following the index * indicates optimal values, while unmarked variables denote all feasible values. The optimal R&D investments have to maximize $H = U(C)+\lambda[F(K, R+\nu b)-C-M(R)B_R(A_R)-M_bB(A)b-I_R-I_b]+\kappa_R I_R+\kappa_b I_b-\tau R$, which gives

$$\begin{aligned}
I_b^* = I_R^* = 0, &\text{ if } -\lambda+\kappa_b<0 \text{ and } -\lambda+\kappa_R<0,\\
0 \le I_b^* \le \bar{I}, &\text{ if } -\lambda+\kappa_b=0 \text{ and } -\lambda+\kappa_R<0,\\
0 \le I_R^* \le \bar{I}, &\text{ if } -\lambda+\kappa_b<0 \text{ and } -\lambda+\kappa_R=0,\\
0 \le I_b^* + I_R^* \le \bar{I}, &\text{ if } -\lambda+\kappa_b=0 \text{ and } -\lambda+\kappa_R=0,\\
I_b^* = \bar{I}, &\text{ if } -\lambda+\kappa_b>0 \text{ and } \kappa_b-\kappa_R>0,\\
I_R^* = \bar{I}, &\text{ if } -\lambda+\kappa_R>0 \text{ and } \kappa_b-\kappa_R<0,\\
I_b^* + I_R^* = \bar{I}, &\text{ if } -\lambda+\kappa_b>0 \text{ and } \kappa_b-\kappa_R=0.
\end{aligned} \tag{11}$$

The optimal research investments are given by (11) together with (9) and the Kuhn-Tucker conditions $\zeta_{\bar{I}} \ge 0$, $\zeta_{\bar{I}}[\bar{I}-I_R-I_b]=0$, $\zeta_{I_b} \ge 0$, $\zeta_{I_b}I_b=0$ and $\zeta_{I_R} \ge 0$, $\zeta_{I_R}I_R=0$. Depending on the relation of κ_b to κ_R and λ as well as of κ_R to λ, the R&D investments in the backstop technology or the exhaustible resource technology are minimal, singular or maximal. E.g.: If $\kappa_b > \lambda$ and $\kappa_b > \kappa_R$, green research

investments are maximal while the black ones are minimal. This implies $\zeta_{I_b} = 0$, $\zeta_{\bar{I}} > 0$ and $\zeta_{I_R} > 0$, such that (9) holds.
The costate variables grow according to

$$\hat{\lambda} = \rho - F_K, \quad (12) \qquad \hat{\tau} = \rho, \quad (13)$$

$$\hat{\kappa}_R = \rho + \frac{\lambda}{\kappa_R} M(R) B'_R, \quad (14) \qquad \hat{\kappa_b} = \rho + \frac{\lambda}{\kappa_b} M_b b B'_b. \quad (15)$$

Differentiating (6) with respect to time and combining with (12) gives

$$\hat{C} = \frac{F_K - \rho}{\eta}. \tag{16}$$

(16) is the well known Ramsey - rule, which states that consumption grows as long as the time preference rate is lower than the marginal product of capital. η is the positive elasticity of marginal utility. Thus, if marginal utility is rather elastic, changes in consumption are small. The transversality conditions

$$(a) \lim_{t\to\infty} e^{-\rho t}\lambda(t)\left[K(t) - K^*(t)\right] \geq 0, \quad (b) \lim_{t\to\infty} e^{-\rho t}\tau(t)\left[S_R(t) - S_R^*(t)\right] \geq 0,$$
$$(c) \lim_{t\to\infty} e^{-\rho t}\kappa_b(t)\left[A_b(t) - A^*(t)\right] \geq 0, \quad (d) \lim_{t\to\infty} e^{-\rho t}\kappa_R(t)\left[A_R(t) - A_R^*(t)\right] \geq 0 \tag{17}$$

complete the equation system of necessary conditions.

3 R as a non-exhaustible resource

Since our model is an extension of Tsur and Zemel (2005), we adapt their analysis method. At every point in time the energy mix is given by (10). Furthermore, (9) and (11) together with the Kuhn-Tucker conditions determine the optimal R&D investments. Therefore, the optimization problem reduces to the task of identifying for all $t \in [0, \infty[$ the optimal consumption and capital investment given optimal R&D and the optimal energy mix. It turns out that this task can be solved by the relative position of four characteristic two dimensional manifolds (planes) in the three dimensional technologies capital (A_b, A_R, K) space. To deduce the characteristic manifolds we assume a non-exhaustible resource stock S_R, i.e. $S_R = \infty$. We will return to the original assumption of a limited resource stock after the derivation of the planes. If $S_R = \infty$, the resource stock is not scarce, implying $\tau = 0$. Thus, (10) becomes

$$F_x(K, x(K, A_b)) = M'(R(A_R, A_b))B_R(A_R) = \frac{M_b}{\nu} B_b(A_b). \tag{18}$$

Since the relative scarcity index does not exist for $S_R = \infty$, capital has no influence on the utilization of the exhaustible resource R. Therefore, (18) determines $x(K, A_b)$, $R(A_R, A_b)$ and $b(K, A_b)$, with $\frac{\partial x}{\partial K} > 0$, $\frac{\partial x}{\partial A_b} \geq 0$, $\frac{\partial R}{\partial A_R} > 0$, $\frac{\partial R}{\partial A_b} \leq 0$, $\frac{\partial b}{\partial K} \geq 0$, $\frac{\partial b}{\partial A_b} \geq 0$ and $\frac{\partial b}{\partial A_R} \leq 0$ as can be seen in Fig. 1 by using comparative statics.
The first three planes are given by the non-arbitrage conditions $\frac{\partial Y^n}{\partial A_b} = \frac{\partial Y^n}{\partial K}$, $\frac{\partial Y^n}{\partial A_R} = \frac{\partial Y^n}{\partial K}$ and $\frac{\partial Y^n}{\partial A_b} = \frac{\partial Y^n}{\partial A_R}$, which compare the effects of green and black research with capital accumulation and with each other. The conditions can be written as

$$-M_b B_b'(A_b) b(K, A_b, A_R) = F_K(K, x(K, A_b)), \tag{19}$$

$$-M(R(A_b, A_R)) B_R'(A_R) = F_K(K, x(K, A_b)), \tag{20}$$

$$-M(R(A_b, A_R)) B_R'(A_R) = -M_b B_b'(A_b) b(K, A_b, A_R). \tag{21}$$

All equations assign maximal one value of K to every A_b, A_R combination. Therefore, all three equations implicitly determine a specific function. These functions are given by $K_b^S(A_b, A_R)$, $K_R^S(A_b, A_R)$ and $K^A(A_b, A_R)$, respectively. In the following we show that every function corresponds with a specific characteristic of the R&D process. In the case of $K_b^S(A_b, A_R)$, we substitute two requirements of singular green research, $\lambda = \kappa_b$ and $\dot{\lambda} = \dot{\kappa}_b$, into (15), which gives $\hat{\lambda} = \rho + M_b B_b(A_b) b$. Using (12) we get (19). Therefore, $K_b^S(A_b, A_R)$ corresponds to singular green research if the black research option is ignored. In a similar manner, $K_R^S(A_b, A_R)$ corresponds to singular A_R research if the A_b research option is ignored. The R&D characteristic corresponding to $K^A(A_b, A_R)$ is given by $\kappa_b = \kappa_R$ and $\dot{\kappa}_b = \dot{\kappa}_R$, i.e. R&D investments are split among both sectors.
All three functions are describing a two-dimensional manifold (plane) in the A_b, A_R, K space. [9] Since the plane described by $K_b^S(A_b, A_R)$ consists of all points of the A_b, A_R, K space where singular A_b research is possible, we call it the singular plane - b (SiP-b). Analogously, the plane described by $K_R^S(A_b, A_R)$ is called SiP-R and the one described by $K^A(A_b, A_R)$ SiP-A. [10]

[9] Although "manifold" is mathematically correct, "plane" is more descriptive. Therefore, we use plane in the following.

[10] To simplify notation, we use the terms SiP-b, SiP-R and SiP-A not only for the planes but also for the functions K_b^S, K_R^S and K^A, respectively.

For every A_b (A_R) level the slope of K_b^S, K_R^S and K^A is given by

$$\frac{dK_b^S}{dA_R} = \frac{M_b B_b' \frac{\partial b}{\partial A_E}}{-\frac{J}{F_{xx}} - M_b B_b' \frac{\partial b}{\partial K}} > 0, \tag{22}$$

$$\frac{dK_b^S}{dA_b} = \frac{F_{Kx}\frac{\partial x}{\partial A_b} + M_b B_b'' b + M_b B_b' \frac{\partial b}{\partial A_b}}{-\frac{J}{F_{xx}} - M_b B_b' \frac{\partial b}{\partial K}}, \tag{23}$$

$$\frac{dK_R^S}{dA_R} = \frac{M' B_R' \frac{\partial R}{\partial A_R} + M B_R''}{-\frac{J}{F_{xx}}}, \tag{24}$$

$$\frac{dK_R^S}{dA_b} = \frac{F_{Kx}\frac{\partial x}{\partial A_b} + M' B_R' \frac{\partial E}{\partial A_b}}{-\frac{J}{F_{xx}}} > 0, \tag{25}$$

$$\frac{dK^A}{dA_R} = \frac{M_b B_b' \frac{\partial b}{\partial A_R} - M B_R'' - M' B_R' \frac{\partial R}{\partial A_R}}{-M_b B_b' \frac{\partial b}{\partial K}}, \tag{26}$$

$$\frac{dK^A}{dA_b} = \frac{M_b B_b'' b + M_b B_b' \frac{\partial b}{\partial A_b} - M' B_R' \frac{\partial R}{\partial A_b}}{-M_b B_b' \frac{\partial b}{\partial K}}. \tag{27}$$

Since the effect of an increasing technology level on $B_b(A_b)$ and $B_R(A_R)$ vanishes for $A_b \to \infty$ and $A_R \to \infty$, respectively, $\frac{dK_b^S}{dA_b}$, $\frac{dK_R^S}{dA_R}$ and $\frac{dK^A}{dA_b}$ are positive for high technology levels, while $\frac{dK^A}{dA_R}$ is negative. To simplify the analysis we follow Tsur and Zemel (2005) and assume monotonic functions, i.e. SiP-b as well as SiP-R increases in both technologies while SiP-A increases in A_b and decreases in A_R. Since the three SiPs correspond with $\kappa_b = \lambda$, $\kappa_R = \lambda$ or $\kappa_b = \kappa_R$, an intersection of only two SiPs is not possible. Together with $\frac{dK_b^S}{dA_R} > 0$, $\frac{dK_R^S}{dA_R}$ and $\frac{dK^A}{dA_R} < 0$ this implies maximal one intersection point along the A_R axis. However, along the A_b axis several intersection points are possible. As $\lim\limits_{A_b \to \infty} \frac{dK^A}{dA_b} = \infty$, the SiP-A lies above SiP-b and SiP-R for large A_b. Thus, the planes form pockets, if several intersection points exist. In the long run these pockets are insignificant, while they complicate the analysis. Therefore, we assume maximal one intersection between the SiPs. Fig. 2 shows the two possible developments with one intersection along the A_b and A_R axis, respectively. It is important to notice that the SiP-A cannot be located between the two others SiPs. Otherwise, the long run development implies an intersection of only two SiPs, which is impossible.

The last of the four planes is given by the Steady State $\hat{A}_b = \hat{A}_R = \hat{K} = \hat{C} = 0$. The technology levels are only constant if no R&D is conducted, which requires $\lambda > \kappa_R, \kappa_b$. Due to (6) a constant consumption implies $\dot{\lambda} = 0$. Substituting into (12) gives

$$F_K(K, x(K, A_b)) = \rho. \tag{28}$$

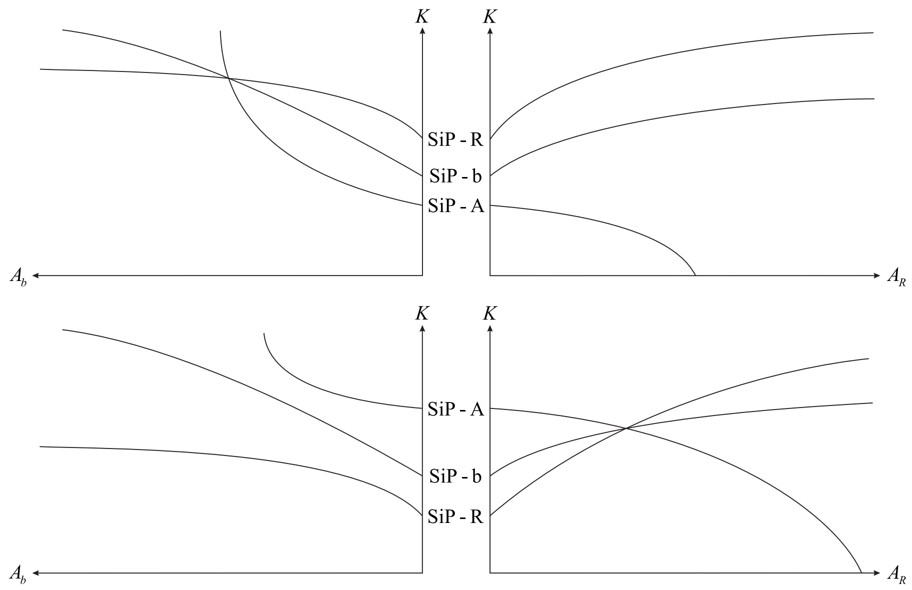

Figure 2: Development of the SiP planes along the technology axes

Since the right hand side is constant, (28) allocates maximal one capital value to every technology combination (A_b, A_R) and therefore implicitly defines the function $K^N(A_b)$. As K_b^S, K_R^S and K^A, the function $K^N(A_b)$ describes a plane in the A_b, A_R, K space. The plane consists of all points where a steady state is possible. Therefore, we call it the steady state plane (SSP). The slope of the SSP is given by $\frac{dK^N}{dA_R} = 0$ and $\frac{dK^N}{dA_b} = -\frac{F_{xx}F_{Kx}}{F_{KK}F_{xx}-F_{Kx}^2}\frac{\partial x}{\partial A_b} \geq 0$. Fig. 3 illustrate the planes in the technologies - capital - space. The line denoted with K_S^S is the intersection line of the three SiP.

In the following we show how the economic development can be predicted by using the four characteristic planes. For this purpose we relate point (K^P, A_b^P, A_R^P) which describes the state of the economy to the planes. We say that "the economy is located above (on, below) the SiP-b, SiP-R, SiP-A or SSP", if $K^P > (<,=)\ i(A_b^P, A_R^P),\ i = K_b^S, K_R^S, K^A, K^N$ holds. The four planes divide the technologies capital space in subspaces with particular static properties:[11]

- Consumption increases (decreases) below (above) the SSP.
- Above (below) the SiP-b the related A_b research has a greater (smaller) effect on the net production than capital accumulation.
- Above (below) the SiP-R the related A_R research has a greater (smaller) effect on the net production than capital accumulation.

[11] See Appendix A.1 for the proof.

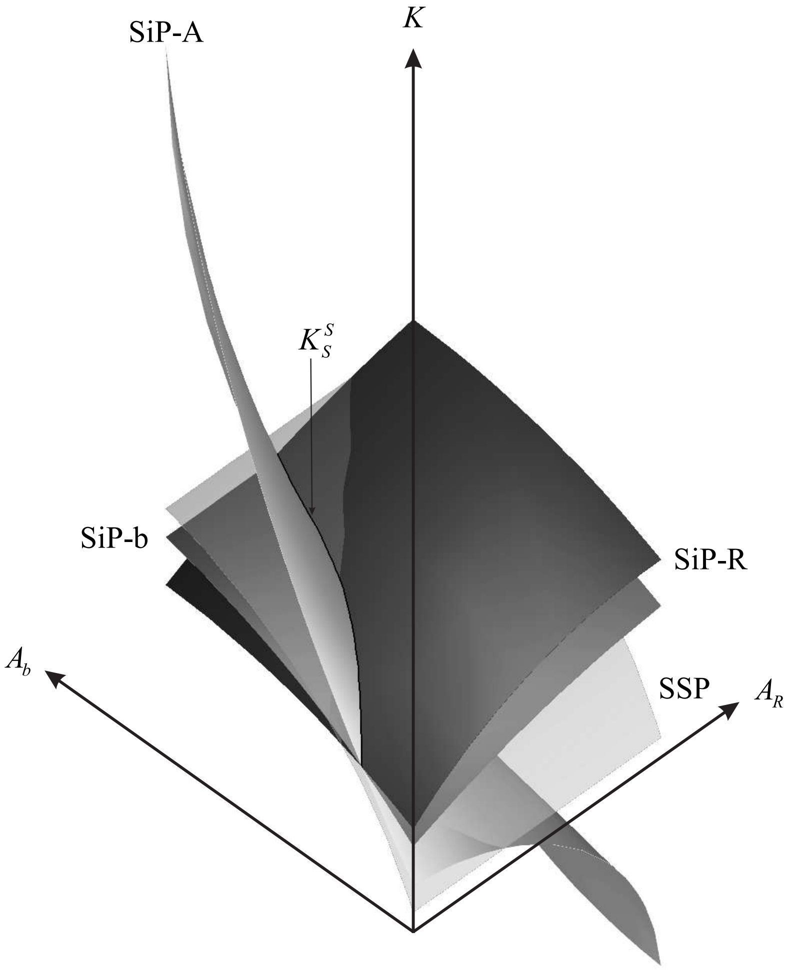

Figure 3: The planes

- Above (below) the SiP-A A_b research has a greater (smaller) effect on the net production than A_R research.

The restrictions of the dynamic development process are derived on the basis of this four properties in Appendix A.2. It is shown that:

i. A development process can be located above the SSP only temporarily.

ii. A steady state above the SiP-b or SiP-R is not optimal.

iii. Maximal A_b (A_R) research investments are only optimal above the SiP-b (SiP-R). Below (on) the SiP-b (SiP-R) the related R&D expenditures are minimal (singular).

iv. All three SiP have a binding effect on the development process.

v. A_b research is not optimal below the SiP-A. A_R research above the SiP-A is only possible, if the development process reaches the SiP-A.

Item i. is explained by $\dot{C} < 0$ above the SSP. If the development path would be located above the SSP permanently, consumption had to decrease. However this can not be optimal, as constant consumption is possible by switching over to the SSP at one point during the process. To understand item ii. recall $\dot{C} > 0$ below the SSP and $\frac{\partial Y^n}{\partial A_i} > \ (=, <) \ \frac{\partial Y^n}{\partial K}$, $i = b, R$ above (on, below) the SiP-b and SiP-R. If the economy is located on the SSP and above the SiP-b (SiP-R), consumption is constant. By converting some capital through research into technology the effect of the increasing technology outweighs the effect of the decreasing capital stock. Therefore, net production increases allowing a higher consumption. The effects of R&D and capital accumulation on net production are also important to rationalize iii. The two extreme R&D options are only possible away from the SiP-b and SiP-R, respectively, as $\frac{\partial Y^n}{\partial A_i} > \frac{\partial Y^n}{\partial K}$, $i = b, R$ above the SiP-b (SiP-R) and $\frac{\partial Y^n}{\partial A_i} < \frac{\partial Y^n}{\partial K}$, $i = b, R$ below it. Only on the both SiP the effects of the related R&D and capital accumulation are equal. Therefore, only SiP-b and SiP-R allow for both an increasing technology and an increasing capital stock. A similar argument is applied on the SiP-A in item v. Above the SiP-A A_b research has a greater effect on net production than A_R research, while the reverse is valid below the SiP-A. However, the argument holds only below and on the SiP-A. Above the SiP-A, it is possible that the less effective A_R research is optimal, as shown below in Fig. 5. In the same manner, capital changes instead of research can be optimal above the SiP-b (SiP-R). Due to item i. to v. we can state:

Proposition 1 *The optimal development process converges at a most-rapid-approach-path either against a steady state on the SSP or the intersection line K_S^S. If K_S^S lies below the SSP for large technology levels, the economy grows for ever along the K_S^S. Otherwise, the economy switches into a steady state on the SSP.*

To understand the most-rapid-approach-path recall that the economy can either conduct minimal or maximal R&D away from the SiP-b or SiP-R. Furthermore, an economy that approaches one of the SiP will reach the intersection line K_S^S at some point in time. Therefore, the intersection line is crucial for defining two economic types which were already distinguished by Tsur and Zemel (2005). On the one hand this is an economy that converges against a steady state (converging economy) and on the other hand an economy with the potential of steady growth (potentially growing economy). The latter is defined by a K_S^S which lies below the SSP for large technology levels, i.e. $\lim_{A_b \to \infty} \lim_{A_R \to \infty} [K_S^S(A_b, A_R) - K^N(A_b, A_R)] < 0$. Analogously, a converging economy is on hand, if the intersection line exceeds the SSP, i.e. $\lim_{A_b \to \infty} \lim_{A_R \to \infty} [K_S^S(A_b, A_R) - K^N(A_b, A_R)] > 0$.

Since a growing economy develops along the K_S^S in the long run, capital increases and research in both sectors is conducted. The simultaneous research in both sectors contrasts sharply with the results of Acemoglu et al. (2012). Although Acemoglu et al. (2012) analyze a decentralized economy, this contradiction highlights the importance of the R&D function. The one of Acemoglu et al. (2012) follows the "building on the shoulders of giants" approach, which assumes a positive research productivity effect of the respective technology level. Therefore, the development process is biased towards the more advanced sector. By shifting more R&D efforts into the advanced sector the productivity wedge between the two sectors increases. Thus, research in the backward sectors becomes less attractive and, in the long run, research will be only conducted in the advanced sector.[12]In the "lab equipment" R&D approach, chosen here, the productivity of research efforts is independent from the technology level, i.e. there does not exists a productivity wedge between the two sectors. On the contrary, the effects of R&D in both sectors on net production equalize, since the impact of R&D in one sector on the related resource costs decreases with a higher technology level. Therefore, a marginal increase of the technology level in the less advanced sector decreases the resource unit costs more than a marginal technology increase in the more advanced sector, making R&D in the backward sector more attractive. Thus, the "lab equipment" approach does not only avoid the productivity wedge but also eliminates a possible impact gap of the two research possibilities on net production. Therefore, R&D expenditures are distributed among both research sectors in the long run.

The optimal development path of a potentially growing economy is derived in Fig. 4. If the capital endowment is sufficiently high ($K_0 > K^4$) the economy approaches one of the SiP with minimal or maximal R&D investments, thus on a MRAP. Due to the bonding force of the SiP, the economy follows the first SiP it reaches. In Fig. 4 an economy with a high capital endowment (K_1^4) conducts maximal A_b research at the beginning. Consequently, the capital stock declines. In the moment the development paths reaches the SiP-A the maximal R&D expenditures are allocated between both research sectors in a manner that guarantees a development along the SiP-A. Following the SiP-A the economy reaches the intersection line K_S^S and switches to singular R&D investments, which are still distributed among both research sectors. However, the capital stock increases now, too. If the capital endowment is only slightly greater than K^4 the economy conducts no research at first and approaches the SiP-b by capital accumulation. As soon as the SiP-b is

[12]Acemoglu et al. (2012) get the same result as we, if both resources are essential for production. In this case the bad substitutability outweighs the productivity effect of a higher technology level.

reached, the economy switches to singular A_b research to follow the SiP-b to the K_S^S, on which the singular R&D expenditures are distributed among both research sectors. In both cases consumption declines at first but growths after the intersection of the development path with the SSP. If the capital endowment is small ($K_0 < K^4$) the research option is too costly for the economy. Therefore, it is ignored and the economy converges against a steady state at the point $(K^N(A_{b_0}), A_{b_0}, A_{R_0})$. This explains the term "potentially" in potentially growing economy, as the economy needs a sufficient high capital endowment to realize the research option. The most pessimistic result appears if the economy is very poor, i.e. the capital endowment below $(K^N(A_{b_0}), A_{b_0}, A_{R_0})$. In this case capital accumulation may be not possible, since the complete production is needed for consumption. Thus, the economy consumes its own capital stock leading to a collapse of production and consumption.

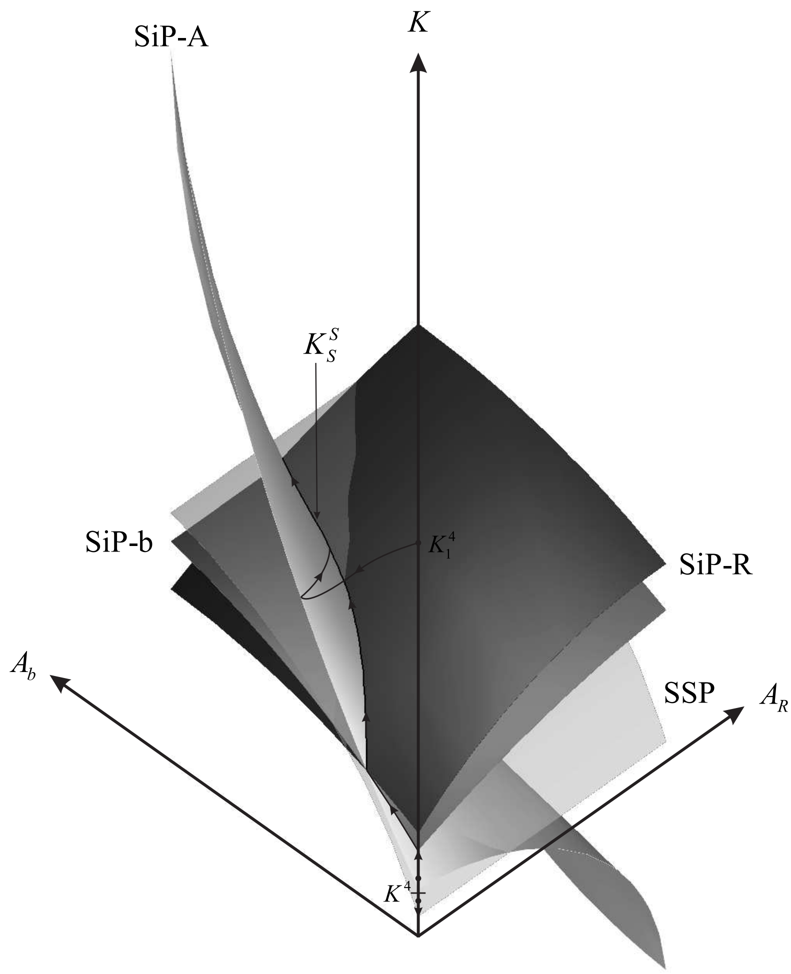

Figure 4: Development paths of a potentially growing economy

Possible development paths of a converging economy are shown in Fig. 5. If the capital endowment is small (K_1^3) the economy approaches the SiP-b or SiP-R by capital accumulation to switch to singular research on one of the SiP and afterwards on the K_S^S. At the intersection of K_S^S and the SSP denoted with S the economy switches into a steady state. With a high capital endowment (K_2^3) the economy will conduct maximal research. Above the SiP-A, A_b is generally more advantageous, so that the economy conducts A_b research as on the path starting at K_2^3. However, A_R research can be the optimal choice above the SiP-A, too. An example is given by the path starting at K_3^3. If A_b research was conducted above the SiP-A, the economy would reach the K_S^S on a point after the intersection point S. This implies declining consumption forever. The alternative path with A_R research leads to the steady state S and is therefore the optimal one. Another possible path above the SiP consists of a part with maximal research which is given up in favor of minimal research before one of the SiP is reached. Such a path converges to a steady state on the SSP with at least one higher technology level than in S.

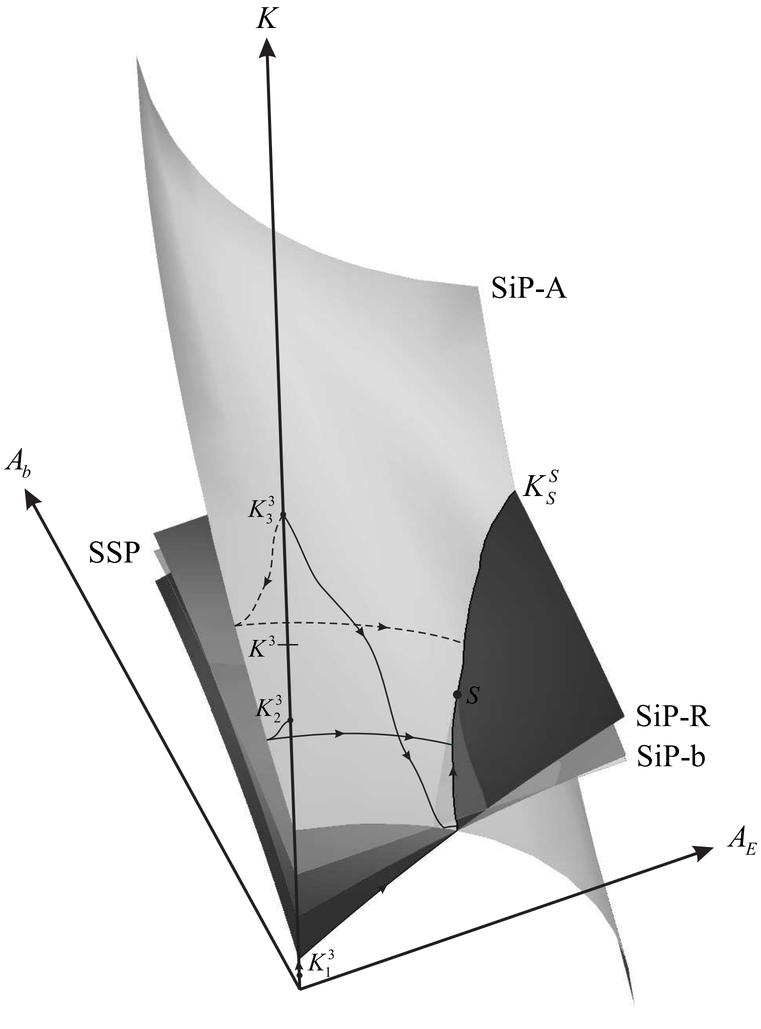

Figure 5: Development paths of a converging economy

We conclude:

Proposition 2 *The economy follows a most rapid approach path to either one of the SiP or the SSP. If the SiP-b or the SiP-R is reached first, the economy switches to the related singular R&D. If the SiP-A is reached first, the maximal research expenditures are divided among the both research sectors. The paths on any of the SiP lead the economy to the intersection line* K_S^S.

4 R as exhaustible Resource

After having derivated the characteristic planes we turn back to the original assumption of an exhaustible resource stock S_R, which implies $\tau > 0$. Equation (10), which determines both the energy supply and the energy mix becomes

$$F_x(K, x(K, A_b, A_R)) = M'(R(K, A_b, A_R))B_R(A_R) + \frac{\tau}{\lambda} = \frac{M_b}{\nu}B_b(A_b). \qquad (29)$$

The scarcity of the resource affects the energy mix through the relative scarcity index $m^q = \frac{\tau}{\lambda}$, which grows according to $\hat{m}^q = F_K$. As can be seen in Fig. 1 a growing scarcity index reduces the input of exhaustible resources under ceteris paribus conditions, while increasing the utilization of backstops. The total energy supply is not affected, since it is independent from the exhaustible resource as long as both resources are used. Because R and b enter (19), (20) and (21), all three SiP are affected by the scarcity. On the other hand, the SSP is defined by (28), which depends on total energy supply. Therefore, the scarcity has no impact on the SSP. The influence of the scarcity on the SiP should be explained by a comparison of the two corner cases $S_R = \infty$ and $S_R = 0$. The case $S_R = 0$ has been analyzed above and establishes the three SiP. Assume the combination K^u, A_b^u, A_R^u is a point on the SiP-b defined for $S_R = \infty$, then $-M_b B_b(A_b^u) b(K^u, A_b^u, A_R^u) = F_K(K^u, x(K^u, A_b^u))$ and $K^u = K_b^S(A_b^u, A_R^u)$. If the resource stock S_R is instead exhausted, energy is only supplied by the backstop, i.e. $x(K^u, A_b^u) = b(K^u, A_b^u, A_R^u) + R(A_R^u, A_b^u) = b^0(K^u, A_b^u) = x^0(K^u, A_b^u)$, with ^{0} denoting the variables in the case of $S_R = 0$. Thus, $b(K^u, A_b^u, A_R^u) < b^0(K^u, A_b^u)$ and therefore $-M_b B_b(A_b^u) b^0(K^u, A_b^u) > F_K(K^u, x(K^u, A_b^u))$. The equation defining the SiP-b is not fulfilled. We now have to determine the capital stock which fulfills the equation given the technology levels A_b^u, A_r^u. A lower capital stock increases the right hand side, because $\frac{dF_K}{dK} = \frac{F_{KK}F_{xx} - F_{Kx}^2}{F_{xx}} < 0$. On the other hand, a lower capital stock reduces energy demand and therefore b, as the marginal product of energy increases in the capital stock. Thus, the left hand side decreases. We conclude that the capital stock must be smaller then K^u. If the consideration is applied to all technology combinations, we get the SiP-b for a exhausted resource stock. Since the capital stock is always smaller, the SiP-b for $S_R = 0$ lies below the SiP-b for $S_R = \infty$ in the technologies capital space. Before using the argumentation for every finite resource stock S_R, we have to notice that every finite resource stock can be associated with a value of the constantly increasing scarcity index m^q and that the utilization of the backstop increases with m^q under ceteris paribus conditions. Then the argumentation establishes a SiP-b for every finite resource stock or every m^q value, respectively, which lies the lower in the A_b, A_R, K space the higher m^q is. By adding the m^q dimension to the technologies capital space, the family of SiP-b forms a three-dimensional manifold in the four-dimensional A_b, A_R, K, m^q space. By projecting the SiP-b on the A_b, A_R, K space, we get a downward shifting plane as shown in Fig. 6, where $^\infty$ and ^{0} denote the SiP-b of the two corner cases. Fig. 6 incorporates also the SSP, which is not affected by scarcity.

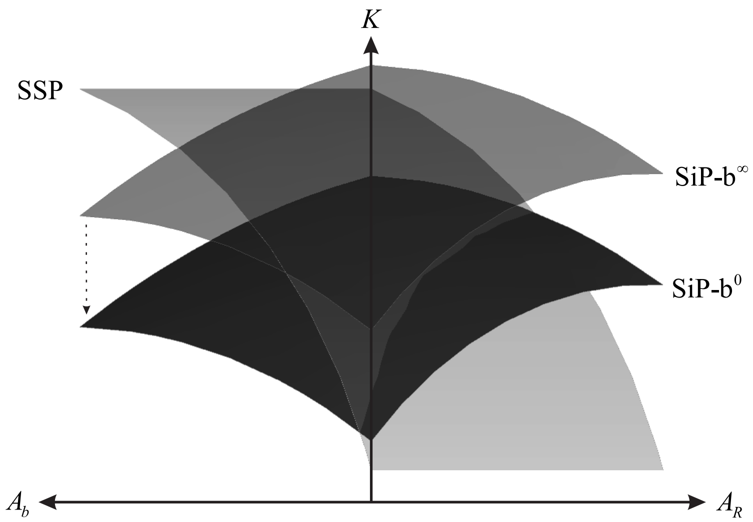

Figure 6: The SiP-b in the A_b, A_R, K space

A similar argumentation is used to analyze the effects of the scarcity on the SiP-R and SiP-A. In case of the SiP-R the scarcity ($\hat{m}^q > 0$) decreases R ceteris paribus and therefore the left hand side of (20): $-M(R(K, A_b, A_R))B'_R(A_R) = F_K(K, x(K, A_b))$. Since the argumentation is based upon a ceteris paribus comparison with a fixed relative scarcity index, different capital stock values do not affect $R(K, A_b, A_R)$. Therefore, the equation holds only for higher capital stock values, because of $\frac{dF_K}{dK} < 0$. In the corner case $S_R = R = 0$, the left hand side is 0 and the equation fulfilled only if $K = \infty$. Thus, the SiP-R is shifting upwards in the A_b, A_R, K space as the relative scarcity index increases.

The SiP-A is given by (21): $-M_b B_b'(A_b) b(K, A_b, A_R) = -M(R(K, A_b, A_R))\, B_R'(A_R)$. An increasing scarcity index reduces the right hand side, while increasing the left hand side. The equation holds only for a reduced capital stock value, since backstop utilization increases in the capital stock while the amount of used exhaustible resources remains unchanged. If the resource stock S_R is exhausted, the right hand side is 0. Energy demand is zero if and only if the capital stock is also zero. Thus, the SiP-A decreases in the A_b, A_R, K space with an increasing scarcity index and is given by the plain spanned by the A_b - and A_R - axis for $S_R = 0$. Fig. 7 illustrate the development of the SiP-R and the SiP-A.

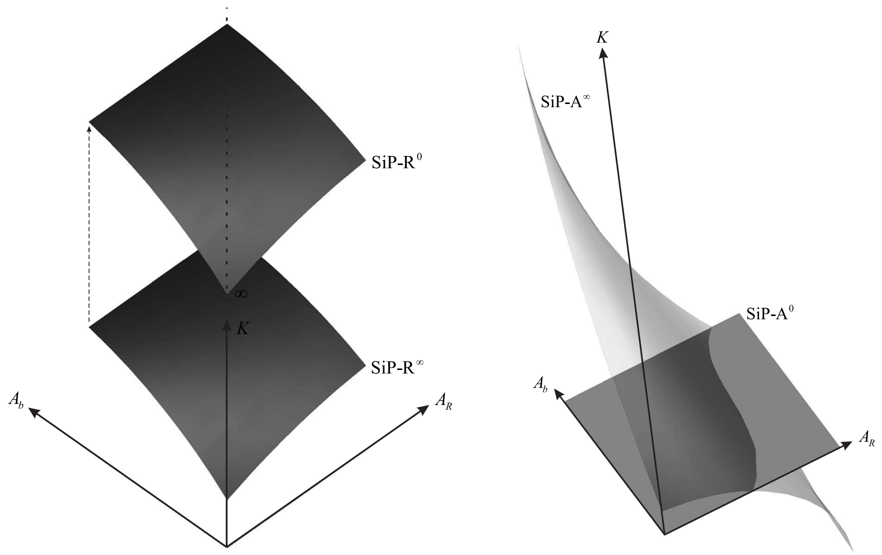

Figure 7: SiP-R and SiP-A in the A_b, A_R, K space

Proposition 3 *The SiP-b (SiP-R, SiP-A) shifts downwards (upwards, downwards) in the A_b, A_R, K space with an increasing m^q. With an exhausted resource stock S_R, SiP-R and SiP-A are only reached for $K = \infty$ and $K = 0$, respectively. The SSP is not affected by the scarcity of R.*

The movement of SiP-b, SiP-R and SiP-A in the technologies capital space can be explained by the scarcity of the exhaustible resource. According to their definition the SiP-b and the SiP-R compare the effect of an increase of A_b and A_R technology, respectively, to a change of the capital stock. It was shown that the capital effect is higher (smaller) below (above) the SiP. Ceteris paribus scarcity reduces the utilization of the exhaustible resource while increases backstop use. Thus, a higher A_b level reduces resource utilization costs more than without scarcity. On the other hand, a higher A_R level has a lower effect on the resource costs. Moreover, the production share of an additional capital unit depends only on total energy input but is independent from the energy mix. Therefore, the scarcity has no influence on the effect of a capital increase. With scarcity the A_b (A_R) technology effect outweighs the capital effect for more (less) technologies - capital combination implying a lower(higher) SiP-b (SiP-R). The SiP-A compares both technology effects to each other. As the A_b effect increases with scarcity and the A_R effect decreases, scarcity raises the number of technologies - capital combination where the A_b effect outweighs the A_R effect. Therefore, the SiP-A lies lower in the technologies capital space.

The long run positions of SiP-R and SiP-A have significant consequences for the development of the economy. The capital value of the SiP-R converges to $K = \infty$ in finite time. However, no economy can accumulate an infinite high capital stock in finite time. Therefore, it is not possible to follow the SiP-R forever. Since the SiP-R bounds the development path, this implies that the optimal path cannot reach the SiP-R. Otherwise, consumption decreases to zero as the whole net production is needed to accumulate capital. Though, a position on the SSP with positive and constant consumption is pareto-superior. Consequently, singular A_R - research is never optimal. Furthermore, as maximal A_R - research is only possible above the SiP-R, maximal A_R - research can be optimal only temporally. This follows also from the long run position of the SiP-A with $K = 0$. As stated above, A_R - research above the SiP-A is only possible, if the development reaches the SiP-A. However, a depleted capital stock implies a production and consumption collapse. In that case any point on the SSP is pareto-superior, since it guarantees a positive and constant consumption. Therefore, a path following the SiP-A, and therefore A_R - research above the SiP-A, can not be optimal. Thus, only A_b - research can be conducted above the SiP-A. This implies that in the long run only A_b - research is possible, as the complete relevant A_b, A_R, K space will lie above the SiP-A. This result is self-explanatory, as the resource stock S_R will be exhausted in finite time. Thereafter, an increase of A_R has no effect on the resource costs and the related research investments must be zero. As black research is not optimal in the long run and the SiP-R and SiP-A are de facto irrelevant, the optimization problem reduces to the one of Tsur and Zemel (2005), i.e. the social planer has to how much net production is invested into green research and/or capital and how much is consumed. Consequently, the definition of the two economic types of a potentially growing and a converging economy must be adapted. Both are no longer defined by the position of the K_S^S in relation to the SSP but by the relation of the SiP-b to the SSP. In the case of a potentially growing (converging) economy the SiP-b lies below (above) the SSP for large technology levels, i.e. $\lim_{A_b \to \infty} K_b^S(A_b, A_R(T_R)) < (>) K^N(A_b, A_R(T_R))$,

with T_R denoting the point in time resource stock S_R gets exhausted.
With the exception of two special cases, which will be described below, the scarcity of R make sure that the three SiP divide the A_b, A_R, K space in subspaces in line with the arbitrage conditions the SiP are based on:

- Above (on, below) the SiP-b $\frac{\partial Y^n}{\partial A_b} > (=, <)\frac{\partial Y^n}{\partial K}$ holds, and maximal (singular, minimal) A_b - research is only optimal above (on, below) the SiP-b.
- Above (below) the SiP-R $\frac{\partial Y^n}{\partial A_R} > (<)\frac{\partial Y^n}{\partial K}$ holds, and maximal (minimal) A_R - research is only optimal above (below) the SiP-R.
- Above (below) the SiP-A $\frac{\partial Y^n}{\partial A_b} > (<)\frac{\partial Y^n}{\partial A_R}$ holds, and A_b - research (A_R - research) is only optimal above (below) the SiP-A.

The first exception appears if green research is optimal below the SiP-A. With an exhaustible resource stock this is possible, since the argumentation from Appendix A.2 Lemma 8(b) does not hold anymore. However, A_R - research above the SiP-A was ruled out. So, if A_b - research is optimal below the SiP-A, green research is generally superior to black research. In that case, the optimization problem of phase 3 reduces to the one discussed by Tsur and Zemel (2005). Therefore, this case is ignored in the following.
The second exception is a converging economy in which research is or becomes generally to expensive. In that case, research is not conducted or abandoned in favor of a steady state reached by variations of capital, even above the SiP-b.
Taken the discussed aspects of scarcity into account Proposition 2 reads now:

Proposition 4 *If the A_b - research is not generally superior to A_R - research, the economy follows a modified MRAP, with superiority of A_b - research (A_R - research) over A_R - research (A_b - research) above (below) the SiP-A, to either the SiP-b or the SSP. On the SiP-b the economy switches to singular A_b - research.*

Our result is in line with the analysis of Acemoglu et al. (2012), who also find that R&D is only conducted in the backstop sector in the long run, but contradicts with the results of Di Maria and Valente (2008). This highlights the importance of the substitutability of the two resource or intermediates. If both intermediates are necessary and the technological progress factor augmenting, as in Di Maria and Valente (2008), it is evident that R&D efforts have to be allocated to the exhaustible resource sector to guarantee a sustainable development. On the other hand, with a high substitutability and a cost reducing technological progress, as in our model, R&D efforts would be wasted, if allocated to the exhaustible resource sector in the long run. Indeed, a sustainable development requires the good substitutability of the two resources, as the technological progress is not factor augmenting. Otherwise, production decreases to zero with the exhaustion of S_R no matter how high the technology levels A_b and A_R are.

The result has also significant consequences for both poor (low capital endowment) and rich (large capital endowment) economies. A poor economy can only conduct research in the area of the backstop resource, although it relies heavily on the exhaustible resource in the short run - a small capital stock implies a low energy demand and therefore a low backstop utilization. This reflects that the final product available for investment is too valuable for a poor economy to be invested in a technology which pays off only temporally. However, in a rich economy (maximal) A_R research may be possible. In this case, the short run resource cost reductions caused by an improving A_R technology outweighs the enduring costs reductions of a better A_b technology. The effect of black research is the higher the higher is the stock S_R. A high S_R implies a long utilization period of the exhaustible resource, in which the improved A_R technology can pay off. In the A_b, A_R, K space this is reflected by a low position of the SiP-R and a high one of SiP-A. However, with a declining resource stock S_R the backstop becomes increasingly important for production, resulting in an increasing attractiveness of A_b research. Due to this effect very rich economies conduct only green research, as they rely heavily on backstops in early periods. The result can also be seen from the resource extraction path. To deduce it, we totally differentiate (29) to obtain the well known Solow-Stiglitz condition

$$\hat{F}_x = \frac{m^q}{M'B_R + m^q} F_K + \frac{M'B_R}{M'B_R + m^q} \left[\hat{B}_R + \frac{M''R}{M'} \hat{R} \right] = \hat{B}_b. \tag{30}$$

(30) is the efficient allocation rule of the exhaustible resource over time. According to the condition the growth rate of the marginal product of energy equals both, the growth rate of the marginal costs of the exhaustible resource and the backstop. If green research is conducted, the latter decreases. As only maximal black research is possible, $\hat{B}_R = 0$ in this case. Therefore, (30) implies $\frac{m^q}{M'B_R} F_K < -\frac{M''R}{M'} \hat{R}$. Since the left hand side is positive, utilization of the exhaustible resource must decline. A similar argumentation holds, if the research efforts are zero. If the research investments are allocated to the black sector, no green research can be conducted. Thus, $\hat{B}_b = 0$ and $\frac{m^q}{M'B_R} F_K = -\hat{B}_R - \frac{M''R}{M'} \hat{R}$. As B_R decreases with improving A_R technology, $-\hat{B}_R > 0$. Therefore, the utilization of the exhaustible resource may increase, if the A_R level is sufficiently enhanced. Due to the finite resource stock S_R the resource utilization cannot grow for ever. On the contrary, an extraction

path with an increasing part must have a peak after which it decreases, implying a higher utilization of the backstop and therefore a higher effect of green research on net production.[13]

5 Pollution

In addition to exhaustible resources, pollution, specifically CO_2 and its effect on global warming, is linked to energy generation. Pollution is extensively discussed in the literature.[14]Following Acemoglu et al. (2012) and Schou (2000), we augment our model with pollution caused by the exhaustible resource. The pollution accumulates in the environment according to

$$\dot{S}_E = R - \gamma S_E. \tag{31}$$

S_E denotes the emission stock, which increases in resource utilization R and decreases due the natural regeneration rate γ.[15]The pollution stock decreases utility that is now given by

$$U = U(C, S_E), \tag{32}$$

with $U_{S_E} < 0$ and $U_{S_E S_E} < 0$. Let θ be the current-value costate variable of the emission stock. As utility decreases, if the emission stock is increased exogenously, the related shadow price θ is negative. Using the same procedure as in the previous section, the necessary condition (7) is now given by

$$\frac{\partial L}{\partial R} = \lambda \left[F_x - M'(R) B_R(A_R) \right] - \tau + \theta = 0. \tag{33}$$

The costate variable develops according to

$$\frac{\partial L}{\partial S_E} = U_{S_E} - \theta\gamma = \rho\theta - \dot{\theta} \tag{34}$$

[13] All variables and functions of (29) determining R are continuous. Therefore, R must be continuous as well. Furthermore, by applying the proof of Tsur and Zemel (2003), Appendix A1 and A2 it is possible to show that the transition form the utilization of both resources to a pure backstop based economy is smooth.

[14] E.g. Bovenberg and Smulders (1995), Chakravorty et al. (2006) and Ricci (2007). A comprehensive overview of the endogenous growth theory related to environmental questions is given by Pittel (2002).

[15] A similar function is used by Acemoglu et al. (2012), Kolstad and Krautkraemer (1993) and Tsur and Zemel (2009).

and the transversality reads

$$\lim_{t\to\infty} e^{-\rho t}\theta(t)\left[S_E(t) - S_E^*(t)\right] \geq 0. \tag{35}$$

Energy input and the energy mix is given by

$$F_x(K, x(K, A_b, A_R)) = M'(R(K, A_b, A_R))B_R(A_R) + \frac{\tau + |\theta|}{\lambda} = \frac{M_b}{\nu}B_b(A_b). \tag{36}$$

A ceteris paribus comparison of (36) with (10) shows that the relative scarcity index is higher with pollution. Therefore, pollution reduces the utilization of the exhaustible resource and boosts backstop. This implies both a lower position of SiP-b and SiP-A and a higher position of SiP-R in the technologies capital space, i.e. a higher attractiveness of A_b research. After the exhaustion of the resource stock S_R at $t = T_R$, the emission stock can not be influenced. Instead, it is determined by $\dot{S}_E = -\gamma S_E$. Therefore, the shadow price θ is zero for all $t > T_R$. To determine the shadow price at $t = T_R$, we split the optimization problem into two parts. In the first part utility is maximized given the known constraints over the time interval $t \in [0, T_R[$, while the problem for the time interval $t \in [T_R, \infty[$ is solved in the second part. The value function of the second part is given by V^S. [16] Then, the shadow price at $t = T_R$ equals $\frac{\partial V^S}{\partial S_E}$.[17]Since a higher emission stock reduces utility U, the value function decreases in S_E, i.e. $\frac{\partial V^S}{\partial S_E} < 0$. This property is important for the forthcoming analysis of a decentralized economy.

Proposition 5 *If the exhaustible resource causes pollution, backstop (exhaustible resource) utilization increases (decreases), which boosts the attractiveness of A_b compared to both capital accumulation and A_R research.*

6 Market Economy

After the analysis of the social optimum we decentralize the economy and show under which conditions the social optimum can be realized in a market economy. With the exception of the energy market we follow the neoclassical assumptions of perfect competition, i.e. the economy consists of a great number of identical individuals and final good producers. The individuals own both capital and the shares of all firms. They buy a composite good, which can be either invested into the capital stock or consumed. The individuals maximize their intertemporal utility

[16] $V^S := \max_{C,b,I_b} \int_{T_R}^{\infty} U(C, S_E(T_R))e^{-\rho[t-T_R]}dt$ subject to $\dot{K} = F(K, b) - C - M_b B_b(A_b)b - I_b$, $\dot{A}_b = I_b$. $0 \leq I_b \leq \bar{I}$ and $b, C, K \geq 0$.

[17] Compare Kamien and Schwartz (2000), page 259 et seqq.

subject to their budget constraint. The composite good producers rent capital from the individuals and purchase energy or resources, respectively, on the energy market. Since they do not face an intertemporal optimization problem, they just maximize their profit at every point in time. Energy is supplied by the resource owners which also conduct research to lower their costs. In the endogenous growth theory it is common to assume that the R&D firms exercise market power due to patents on new developments. We apply this assumptions to our framework such that there are two R&D firms, one in the green and one in the black sector. As research is conducted by the resource owners, the backstop as well as the exhaustible resources are each owned by one firm. [18] For the market structure we assume a Cournot competition. [19]To conduct research the firms have to buy goods on the composite good market. The government levies the unit tax ϕ on the exhaustible resource and subsidizes the utilization of both resources with s_b and S_R, respectively. A balanced budget is guaranteed by the lump-sum transfer $T \lesseqgtr 0$.

The representative individual maximizes $\int_0^\infty U(C, S_E)e^{-\rho t}dt$ subject to $\dot{K} = \frac{r}{p_Y}K + \frac{\pi}{p_Y} + \frac{\pi_b}{p_Y} + \frac{\pi_R}{p_Y} + \frac{T}{p_Y} - C$, with r denoting the interest rate, p_Y the composite good price and π, π_b and π_R the profits of the composite good producers, the backstop resource supplier and the exhaustible resource supplier, respectively. Due to the great number of individuals, a single individual takes the profits, the transfer and the emission stock as exogenously given. With the costate variable λ_H the current value Hamiltonian is given by $H = U(C, S_E) + \lambda_H \left[\frac{r}{p_Y}K + \frac{\pi}{p_Y} + \frac{\pi_b}{p_Y} + \frac{\pi_R}{p_Y} + \frac{T}{p_Y} - C\right]$. The composite good producers rent their capital from the individuals and buy the energy from the resource owners. They maximize their profit for every point in time. The representative producer's profit is given by $\pi = p_Y F(K, R+\nu b) - rK - p_b b - (p_R + \phi)R$, with p_b and p_R denoting the prices of energy generated by means of backstops and exhaustible resources, respectively. The first order conditions of the optimization problems establish the resource demand function $p_Y F_x(K, R+\nu b) = \frac{p_b}{\nu} = p_R + \phi$ and the Ramsey - rule, as stated in the third column of Table 1.

The resource owners maximize their intertemporal profit subject to the development of the related technology and in case of the exhaustible resources subject to the limited resource stock. Since the resource owners are engaged in a Cournot com-

[18]It is possible to assume more detailed R&D sectors with several firms, as in Acemoglu et al. (2012). However, this increases the complexity of the model considerably without gaining more insight. Finally, the government has to internalize market power effects stemming from the monopoly of R&D firms concerning a specific technology, machine, etcetera.

[19]The assumption of a Cournot competition is made to allow for positive R&D investments in the green sector without governmental intervention. With Bertrand competition the earnings of the backstop owners would equal their costs. Therefore, green research investments must be directly given by the government.

petition, they take the resource demand function into account. The instantaneous profits are given by

$$\pi_b = p_Y F_x(K, R + \nu b)\nu b - p_Y M_b B_b(A_b)b + s_b b - p_Y I_b, \tag{37}$$

$$\pi_R = [p_Y F_x(K, R + \nu b) - \phi]R - p_Y M(R) B_R(A_R) + s_R R - p_Y I_R. \tag{38}$$

With κ_{bH}, κ_{RH} and τ_H as costate variables the current value Hamiltonians are given by $H_b = p_Y F_x(K, R + \nu b)\nu b - p_Y M_b B_b(A_b)b + s_b b - p_Y I_b + \kappa_{bH} I_b$ and $H_R = [p_Y F_x(K, R + \nu b) - \phi]R - p_Y M(R) B_R(A_R) + s_R R - p_Y I_R - \tau_H R + \kappa_{RH} I_R$. In case of the backstop owner, the first order conditions can be written as[20]

$$F_x(K, R + \nu b) = \frac{M_b}{\nu} B_b(A_b) - F_{xx}(K, R + \nu b)\nu b - \frac{s_b}{\nu p_Y}, \tag{39}$$

$$\hat{\kappa}_{bH} = \rho + \frac{p_Y}{\kappa_{bH}} M_b B_b'(A_b) b. \tag{40}$$

For the exhaustible resource owner we get the first order conditions[21]

$$F_x(K, R + \nu b) = M'(R) B_R(A_R) - F_{xx}(K, R + \nu b)R + \frac{\phi}{p_Y} - \frac{s_R}{p_Y} + \frac{\tau_H}{p_Y}, \tag{41}$$

$$\hat{\kappa}_{RH} = \rho + \frac{p_Y}{\kappa_{RH}} M(R) B_R(A_R). \tag{42}$$

and $\hat{\tau}_H = \tau_{0H} e^{\rho t}$, with τ_{0H} as the initial value of τ_H. The capital stock, the tax, the subsidies and the price p_Y are taken by the energy suppliers as given, while they determine the related technology by their own decision. The development τ_H depends only on its initial value. Therefore, (39) and (41) define implicitly optimal resource utilization subject to the resource supply of the competitor and the technology level, i.e. $b^*(R, A_b)$ and $R^*(b, A_R)$. The Nash - Cournot equilibrium is established by substituting $b^*(R, A_b)$ and $R^*(b, A_R)$ into (41) and (39), respectively:

$$\begin{aligned} F_x(K, R^* + \nu b^*(R^*, A_b)) =& M'(R^*) B_R(A_R) - F_{xx}(K, R^* + \nu b^*(R^*, A_b))R^* \\ &+ \frac{\phi}{p_Y} - \frac{s_R}{p_Y} + \frac{\tau_H}{p_Y}, \\ F_x(K, R^*(b^*, A_R) + \nu b^*) =& \frac{M_b}{\nu} B_b(A_b) - F_{xx}(K, R^*(b^*, A_R) + \nu b^*)\nu b^* - \frac{s_b}{\nu p_Y}. \end{aligned}$$

[20] The transversality condition is $\lim_{t \to \infty} e^{-\rho t} \kappa_{bH} [A_b(t) - A_b^*(t)] \geq 0$.

[21] The transversality conditions are $\tau_H(T_R) = \gamma_{S_R}$, $\gamma_{S_R} \geq 0$, $\gamma_{S_R} S_R(T_R) = 0$ and $H_R(T_R) = \begin{cases} \leq 0, & \text{if} T_R = 0 \\ = 0, & \text{if} 0 < T_R < \infty \\ \geq 0, & \text{if} T_R = \infty \end{cases}$.

Finally, the maximization of the two Hamiltonians with respect to research investments gives

$$\begin{aligned} I_b^* &= 0, \text{ if } -p_Y + \kappa_{bH} < 0, & I_R^* &= 0, \text{ if } -p_Y + \kappa_{RH} < 0, \\ 0 \le I_b^* &\le \bar{I} - I_R, \text{ if } -p_Y + \kappa_{bH} = 0, & 0 \le I_R^* &\le \bar{I} - I_b, \text{ if } -p_Y + \kappa_{RH} = 0, \\ I_b^* &= \bar{I} - I_R, \text{ if } -p_Y + \kappa_{bH} > 0, & I_R^* &= \bar{I} - I_b, \text{ if } -p_Y + \kappa_{RH} > 0. \end{aligned} \tag{43}$$

A comparison of the second and the third column of Table 1 reveals that the social optimum can be realized in a decentralized economy, if the exhaustible resource is taxed and both resources subsidized according to Proposition 6

Proposition 6 *In the case the resource owner with the higher willingness to pay for R&D dominates the other resource owner on the composite good market, the market equilibrium replicates the social optimum, if $\lambda_H = \lambda$, $\tau_H = \tau$, $\kappa_{bH} = \kappa_b$ and $\kappa_{RH} = \kappa_R$ hold, the resources are subsidized according to $s_b = -p_Y F_{xx} \nu^2 b$ and $s_R = p_Y F_{xx} R$, respectively, and the exhaustible resource tax equals $|\theta|$.*

See Appendix A.3 for the proof.

The optimal development of the tax is given by (34):

$$\hat{\phi} = (\rho + \gamma)\phi + U_{S_E} \tag{44}$$

As the first term on the right hand side is positive and the second negative, the tax may increase as well as decrease. To analyze the effects of R&D on the tax, we totally differentiate (36)

$$\left[\hat{\phi} - \rho\right] \frac{\phi}{\tau + \phi} + F_K = \frac{B_R M' + \frac{\tau+\phi}{\lambda}}{\frac{\tau+\phi}{\lambda}} \hat{B}_b - \frac{B_R M'}{\frac{\tau+\phi}{\lambda}} \left[\hat{B}_R + \frac{M'' R}{M'} \hat{R}\right]. \tag{45}$$

The left hand side is the growth rate of the relative scarcity index $\frac{\tau+\phi}{\lambda}$, while the right hand side shows the three forces that determine the development of the index. The tax decreases in both green research and the extraction growth rate $\hat{R}$, whereas it increases in black research. The first point is explained by the backstop costs reduction, which boosts the utilization of the backstop and depresses the extraction of exhaustible resources. Therefore, less pollution is emitted, implying a lower tax. The opposite argument holds for black research, which decreases the costs of the exhaustible resources. This boosts black energy and therefore pollution, implying a higher tax. The second point shows how the government can influence the resource extraction by the tax. E.g. assume minimal R&D, then a higher extraction rate needs a lower tax. Since research can be only conducted in one sector and the black

	social optimum	decentralized economy
Ramsey - rule	$\hat{C} = \frac{F_K - \rho}{\eta}$	$\hat{C} = \frac{F_K - \rho}{\eta}$
marginal product of R	$F_x = M'(R)B_R(A_R) + \frac{\tau - \theta}{\lambda}$	$F_x = M'(R)B_R(A_R) - F_{xx}R + \frac{\phi}{p_Y} - \frac{s_R}{p_Y} + \frac{\tau_M}{p_Y}$
marginal product of b	$F_x = \frac{M_b}{\nu}B_b(A_b)$	$F_x = \frac{M_b}{\nu}B_b(A_b) - F_{xx}\nu b - \frac{s_b}{\nu p_Y}$
capital accumulation	$\dot{K} = F(K,x) - M_bB(A)b - M(R) - I - C$	$\dot{K} = F(K,x) - M_bB(A)b - M(R) - I - C$
R&D	$\hat{\kappa}_R = \rho + \frac{\lambda}{\kappa_R}M(R)B'_R(A_R)$ $\hat{\kappa}_b = \rho + \frac{\lambda}{\kappa_b}M_bB'_b(A_b)b$ (i) $I_b^* = I_R^* = 0$, if $-\lambda + \kappa_b < 0$ and $-\lambda + \kappa_R < 0$ (ii) $0 \leq I_R^* \leq \bar{I}$, if $-\lambda + \kappa_b < 0$ and $-\lambda + \kappa_R = 0$ (iii) $0 \leq I_b^* \leq \bar{I}$, if $-\lambda + \kappa_b = 0$ and $-\lambda + \kappa_R < 0$ (iv) $0 \leq I_b^* + I_R^* \leq \bar{I}$, if $-\lambda + \kappa_b = 0$ and $-\lambda + \kappa_R = 0$ (v) $I_b^* = \bar{I}$, if $-\lambda + \kappa_b > 0$ and $\kappa_b - \kappa_R > 0$ (vi) $I_R^* = \bar{I}$, if $-\lambda + \kappa_R > 0$ and $\kappa_b - \kappa_R < 0$ (vii) $I_b^* + I_R^* = \bar{I}$, if $-\lambda + \kappa_b > 0$ and $\kappa_b - \kappa_R = 0$	$\hat{\kappa}_{RH} = \rho + \frac{p_Y}{\kappa_{RH}}M(R)B'_R(A_R)$ $\hat{\kappa}_{bH} = \rho + \frac{p_Y}{\kappa_{bH}}M_bB'_b(A_b)b$ (I) $I_R^* = 0$, if $-p_Y + \kappa_{RH} < 0$ (Ia) $I_b^* = 0$, if $-p_Y + \kappa_{bH} < 0$ (II) $0 \leq I_R^* \leq \bar{I} - I_b$, if $-p_Y + \kappa_{RH} = 0$ (III) $0 \leq I_b^* \leq \bar{I} - I_R$, if $-p_Y + \kappa_{bH} = 0$ (V) $I_b^* = \bar{I} - I_R$, if $-p_Y + \kappa_{bH} > 0$ (VI) $I_R^* = \bar{I} - I_b$, if $-p_Y + \kappa_{RH} > 0$

Table 1: Comparison of the decentralized economy and the social optimum

research option can be only realized by rich economies, (45) connotes that rather rich than poor economies should exhibit a high tax rate on fossil fuels.

7 Conclusion

The literature regarding directed technical change and exhaustible resources exhibit two critical aspects if applied on energy generation: resource augmenting technological progress and the absence of backstops. This paper analyzes the direction of energy related technological progress without these two critical aspects. On the one hand, it incorporates backstops. On the other hand, it applies a cost reducing technological progress instead of a resource augmenting one. Both is realized by the augmentation of Tsur and Zemel (2005) with a second technology. We show that research in the exhaustible resource sector can be optimal only temporary. This result, which differs totally from Di Maria and Valente (2008), highlights the importance of the substitutability of the exhaustible resource and the form of the technological progress. If the technological progress is factor augmenting, a sustainable development is possible even if the exhaustible is necessary for production. By abandon this assumption and introducing cost reducing technological progress, we show that a good substitutability of the exhaustible resource is necessary for a sustainable development. If those two energy sources are bad substitutes, production collapses in the moment when the resource stock becomes exhausted.

Since the exhaustible resource cannot used forever, a reduction of its extraction costs is of temporary nature. Due to the availability of a backstop, the research inputs are then too valuable for a poor economy, i.e. an economy with a low capital endowment, to be invested in this cost reduction. Thus, despite the heavy reliance on the exhaustible resource in early periods, black research is not an option for poor economies. Instead the composite goods are consumed or invested either in capital or in green research. These trade-offs between the two research options as well as between both research and capital accumulation and consumption and investments are illustrated by four characteristic manifolds. In a rich economy, i.e. an economy with a sufficiently high capital endowment, the cost reduction can be high enough to justify research in the exhaustible resource sector. However, with a declining resource stock the attractiveness of black research compared with both capital accumulation and green research decreases and vanishes completely with the exhaustion of the resource. On the other hand, the attractiveness of backstop research is boosted by the increasing share of the backstop in the energy mix. Thus, in the long run a rich economy can also only conduct green research. If and for how long research

is conducted is determined by two of the four manifolds, the steady state and the singular-b plane, and the capital endowment of the economy, resembling the results of Tsur and Zemel (2005). Accordingly, an economy which realizes the research option either grows forever or switches into a steady state with a technology level higher then its endowment level.
In the case that the exhaustible resource causes CO_2 - emissions the effect of scarcity on R&D is enforced by a pollution effect. As the utilization of the exhaustible resource harms the environment, it is optimal to use lesser exhaustible resources than without pollution. Consequently, the utilization of the backstop increases, implying a higher attractiveness of green research compared with both black research and capital accumulation.
To realize the social optimum in a decentralized economy the government has to subsidize both resources to counter market power effects of the research firms, which are common among endogenous growth models. Additionally, the exhaustible resource must be taxed if it pollutes. The tax increases in black research while it decreases in green research. Since the former can be only conducted by rich economies, our model suggests that an emission tax should be the higher the richer the economy.

By applying the "lap equipment" R&D approach instead of the "standing on the shoulders of giants" one, used by Acemoglu et al. (2012), we emphasize the importance of the assumption related to the research sectors. If both resources are non-exhaustible, the approach of Acemoglu et al. (2012) drives a wedge between the productivity of the research sectors. Therefore, R&D is only conducted in the more advanced sector in the long run. Our approach does not allow for such a productivity wedge. Instead it eliminates inequalities of the R&D effects. Consequently, we find that the R&D efforts are split among both sectors in the long run, which contrasts completely with the result if one resource is exhaustible.

We have already augmented our model with pollution and environmental concerns in 5. Further extensions in this area, like a ceiling on the stock of pollution, seems to be an interesting field of research.

References

Acemoglu, D., 1998. Why do new technologies complement skills? Directed technical change and wage inequality. The Quarterly Journal of Economics 113 (4), 1055–1089.

Acemoglu, D., 2002. Directed technical change. Review of Economic Studies 69 (4), 781–809.

Acemoglu, D., Aghion, P., Bursztyn, L., Hemous, D., 2012. The Environment and directed technical change. American Economic Review 102 (1), 131–166.

Aghion, P., Howitt, P., García-Peñalosa, C., 1998. Endogenous growth theory. the MIT Press.

Barbier, E., 1999. Endogenous growth and natural resource scarcity. Environmental and Resource Economics 14 (1), 51–74.

Barro, R., Sala-i Martin, X., 2003. Economic growth, 2nd Edition. The MIT Press.

Bovenberg, A., Smulders, S., 1995. Environmental quality and pollution-augmenting technological change in a two-sector endogenous growth model. Journal of Public Economics 57 (3), 369–391.

Chakravorty, U., Magné, B., Moreaux, M., 2006. A Hotelling model with a ceiling on the stock of pollution. Journal of Economic Dynamics and Control 30 (12), 2875–2904.

Dasgupta, P., Heal, G., 1974. The optimal depletion of exhaustible resources. The Review of Economic Studies 41, 3–28.

Di Maria, C., Valente, S., 2008. Hicks meets Hotelling: the direction of technical change in capital–resource economies. Environment and Development Economics 13 (06), 691–717.

Grimaud, A., Rouge, L., 2008. Environment, directed technical change and economic policy. Environmental and Resource Economics 41 (4), 439–463.

Hartwick, J., 1977. Intergenerational equity and the investing of rents from exhaustible resources. The American Economic Review 67 (5), 972–974.

Kamien, M., Schwartz, N., 2000. Dynamic optimization: the calculus of variations and optimal control in economics and management, 2nd Edition. North-Holland New York.

Kiley, M., 1999. The Supply of Skilled Labour and Skill-biased Technological Progress. The Economic Journal 109 (458), 708–724.

Kolstad, C., Krautkraemer, J., 1993. Natural resource use and the environment. Handbook of Natural Resource and Energy Economics 3, 1219–1265.

Meadows, D., Meadows, D., Randers, J., Behrens, W., of, R. C., 1972. The limits to growth. Universe Books New York.

Meyer, E., Müller-Siebers, K., Ströbele, W., 1998. Wachstumstheorie. Oldenbourg.

Pittel, K., 2002. Sustainability and endogenous growth. Edward Elgar Publishing.

Ricci, F., 2007. Environmental policy and growth when inputs are differentiated in pollution intensity. Environmental and Resource Economics 38 (3), 285–310.

Rivera-Batiz, L., Romer, P., 1991. Economic integration and endogenous growth. The Quarterly Journal of Economics 106 (2), 531–555.

Schou, P., 2000. Polluting non-renewable resources and growth. Environmental and Resource Economics 16 (2), 211–227.

Solow, R., 1974. Intergenerational equity and exhaustible resources. The review of economic studies 41, 29–45.

Stiglitz, J., 1974. Growth with exhaustible natural resources: efficient and optimal growth paths. The review of economic studies 41 (128), 123–137.

Tsur, Y., Zemel, A., 2003. Optimal transition to backstop substitutes for nonrenewable resources. Journal of Economic Dynamics and Control 27 (4), 551–572.

Tsur, Y., Zemel, A., 2005. Scarcity, growth and R&D. Journal of Environmental Economics and Management 49 (3), 484–499.

Tsur, Y., Zemel, A., 2009. Endogenous discounting and climate policy. Environmental and Resource Economics 44 (4), 507–520.

Tsur, Y., Zemel, A., 2011. On the dynamics of competing energy sources. Automatica.

A Appendix

As the basic framework of the model is based on Tsur and Zemel (2005), the following Appendix resembles their one.

A.1 Properties of the subspaces

On the SSP $F_K = \rho$ must hold. If the capital stock is increased (decreased) exogenously, a point above (below) the SSP is reached. As the marginal product of capital decreases in K, $F_K < \rho$ ($F_K > \rho$) holds above (below) the SSP. According to Ramsey - rule (16) this implies a decreasing (increasing) consumption.

Lemma 1 *Consumption increases below the SSP and decreases above it.*

To evaluate the effects of capital accumulation and R&D we define $\Lambda_b := Y_K^n - Y_{A_b}^n$ and $\Lambda_R := Y_K^n - Y_{A_R}^n$. Due to the definition of the SiP-b and SiP-R $\Lambda_b(K_b^S(A_b, A_R), A_b, A_R) = 0$ holds on the SiP-b and $\Lambda_R(K_R^S(A_b, A_R), A_b, A_R) = 0$ holds on the SiP-R. The reaction of Λ_b and Λ_R to changes of the capital stock are given by:

$$\frac{d\Lambda_b}{dK} = \frac{J}{F_{xx}} + B_b' M_b \frac{\partial b}{\partial K} < 0 \tag{46}$$

$$\frac{d\Lambda_R}{dK} = \frac{J}{F_{xx}} < 0 \tag{47}$$

Therefore, a higher capital stock decreases both Λ_b and Λ_R, implying that the effect of a technology improvement on net production outweighs the one of a higher capital above both SiP, while below the SiP the effect of the capital stock increase is bigger.

Lemma 2 *Above both the SiP-b and the SiP-R an increase of the related technology has a higher effect on net production than an increase of the capital stock. Below the SiP-b and SiP-R the opposite holds. The effects are equal on the SiP-b and the SiP-R, respectively.*

A similar argument can be applied on the SiP-A. For this purpose we define $\Lambda_A := Y_{A_R}^n - Y_{A_b}^n$. On the SiP-A $\Lambda_A(K^A(A_b, A_R), A_b, A_R) = 0$ holds. A higher capital stock decreases Λ_A, since $\frac{\partial \Lambda_A}{\partial K} = M_b B_b'(A_b) \frac{\partial b}{\partial K} < 0$. Thus, we get:

Lemma 3 *Above the SiP-A an increase of backstop technology has a higher effect on net production than an increase of A_R. Below the SiP-A the opposite holds. The effects are equal on the SiP-A.*

A.2 The development process

To deduce the influence of Λ_b and Λ_R on the R&D investments we define $\varsigma_i := \kappa_i - \lambda,\ i = b, R$. Minimal (singular, maximal) research in the related sector requires $\varsigma_i < (=, >)0$. Subtracting (12) from (15) and (14), respectively, gives

$$\hat{\varsigma}_b = \rho + \frac{\lambda}{\varsigma_b}\Lambda_b \tag{48}$$

$$\hat{\varsigma}_R = \rho + \frac{\lambda}{\varsigma_R}\Lambda_R \tag{49}$$

Assume the economy is located below the SiP-b and R&D investments are maximal, i.e. $\varsigma_b > 0$ and $\Lambda_b > 0$. (48) implies then $\lim\limits_{t\to\infty} e^{-\rho t}\varsigma_b(t) = \infty$ and due to $\lambda > 0$ also $\lim\limits_{t\to\infty} e^{-\rho t}\kappa_b(t) = \infty$. However, this violates the transversality conditions (17)(c). Since $\frac{dK_b^S}{dA_b} > 0$ and maximal R&D allows only $\dot{K} \leq 0$, the SiP-b can not be reached by maximal A_b research below the SiP-b. Therefore, a development process with this property violates the transversality condition and is not optimal. A similar argument holds for maximal A_R research below the SiP-R. Since singular research investments require a position on either the SiP-b or the SiP-R, we can conclude:

Lemma 4 *Maximal A_b or A_R research investments are only possible above the SiP-b or the SiP-R, respectively.*

The opposite case is minimal R&D above the SiP-b. In this case, $\varsigma_b < 0$ and $\Lambda_b < 0$, implying $\lim\limits_{t\to\infty} e^{-\rho t}\varsigma_b(t) = -\infty$. Since $\kappa_b \geq 0$, $\lim\limits_{t\to\infty} e^{-\rho t}\lambda(t) = \infty$, violating transversality condition (17)(a). Since one property of a steady state is $\dot{A}_b = 0$, a steady state can not be located above the SiP-b. The argument can be applied on the A_R research sector in a similar manner.

Lemma 5 *A steady state can not be located above both the SiP-b and the SiP-R.*

Minimal R&D is also conceivable for two other economic situations. In the first one the economy is located between the SiP-b and the SiP-R and in the second one below both. As shown above, an economy with minimal R&D can not stay above the SiP-b or the SiP-R forever. Since research is generally too expensive in this case, the development path must approach the SSP, which lies below both SiP, by means of capital reduction. If the economy is located between the SiP-b and the SiP-R, the development path can either approach the upper SiP by capital accumulation or the SSP. In the latter case, R&D is not conducted at all. In the first case, the economy switches to singular R&D investments in the respective sector once the upper SiP is reached. The last option is a position below both the SiP-b and the SiP-R. The economy can approach either one of the SiP, implying singular R&D afterwards, or the SSL, implying no R&D at all.

Lemma 6 *With minimal R&D the economy can approach the SiP-b, the SiP-R or the SSP, depending on its position in the A_b, A_R, K space. If located above both the SiP-b and the SiP-R, only a convergence against the SSP below both SiP is possible. If located between the SiP-b and SiP-R, the economy can either approach the upper SiP or the SSP below both SiP. If located below both SiP, the development can either lead to one of the SiP or the SSP below both SiP. If one of the SiP is reached, appropriate singular R&D investments are adopted.*

If an development path has once reached the SiP-b (SiP-R), $\Lambda_b = 0$ ($\Lambda_R = 0$) holds. A divergence from the SiP-b (SiP-R) implies $\Lambda_b \lesseqgtr 0$ ($\Lambda_R \lesseqgtr 0$). If the capital stock exceeds K_b^S (K_R^S) at the moment of divergence, $\varsigma_b < 0$ ($\varsigma_R < 0$), because $\Lambda_b < 0$ ($\Lambda_R < 0$). This implies minimal R&D investments above the SiP-b (SiP-R). However, the singular R&D investments before the divergence were not optimal in this case. In the opposite case, i.e. the capital stock deceeds K_b^S (K_R^S) at the moment of divergence, the argument implies maximal R&D investments below the SiP-b (SiP-R), which is not optimal according to lemma 4. Therefore, the SiP-b and SiP-R exhibit a binding effect with respect to the development process.

Lemma 7 *On the SiP-b and SiP-R singular R&D is either conducted forever or abandoned if the economy switches to a steady state at the intersection of SiP-b or SiP-R, respectively, and the SSP.*

Analogously to ς_b and ς_R we define $\varsigma_A := \kappa_E - \kappa_b$. (15) and (14) give

$$\hat{\varsigma}_A = \rho - \frac{\lambda}{\varsigma_A}\Lambda_A. \tag{50}$$

A situation with the optimality of A_b (A_R) research below (above) the SiP-A is characterized by $\varsigma_A <$ (>) 0 and $\Lambda_A >$ (<) 0. If an economy is permanently located below (above) the SiP-A, $\hat{\varsigma}_A <$ (>) 0 implies $\lim_{t\to\infty} e^{-\rho t}\kappa_b(t) = \infty$ ($\lim_{t\to\infty} e^{-\rho t}\kappa_R(t) = \infty$), violating transversaility condition (17) (c) ((d)). At first, we assume that R&D is generally too expensive and R&D minimal. Then the development process must approach the intersection of SiP-A and SSP below the SiP-b and SiP-R. If R&D are either singular or maximal, the development process is located on the SiP-b or SiP-R, respectively, or approaches one of the two SiP. In the case that the SiP concerned is the SiP-b, the economy is located below the SiP-A forever, since the SiP-A exceeds the SiP-b and SSP for large A_b and we assumed that the economy must be located below the SiP-A but above the SiP-b to conduct A_b research. Because the transversality condition (17) (c) is violated in this case, A_b below the SiP-A is not optimal. In the case of an economy located above the SiP-A, the development process can reach the SiP-A by means of singular or maximal A_R research as long as the capital stock is sufficiently small. If the capital stock is high, the intersection of SSP and SiP-A can be reached by switching form maximal A_R research to minimal research.

Lemma 8 *(a) A_R research above the SiP-A can be only optimal if the development process reaches either the SiP-A by means of maximal or singular research or the intersection of SiP-A and SSP below SiP-b and SiP-R with minimal research. (b) A_b research below the SiP-A is never optimal.*

On the SiP-A $\Lambda_A = \varsigma_A = 0$ holds. If an economy with singular or maximal R&D divergences from the SiP-A with $K < K^A$, $\Lambda_A > 0$ and therefore $\varsigma_A < 0$ at the moment of divergence. This implies the non-optimal A_b research below the SiP-A. In the case of $K > K^A$ research in the A_R sector ist optimal above the SiP-A. However, this forces the development path back on the SiP-A.

Lemma 9 *A development process can not leave the SiP-A downwards and only temporarily upwards.*

Lemma 1 implies that an economy can not be located permanently above the SSL, since consumption would decrease forever while a point on the SSP guarantees constant consumption.

Lemma 10 *A development path is not optimal if located above the SSP permanently.*

A.3 The social optimum and the decentralized economy

Ramsey - rule of the social optimum and the decentralized economy is identical. By substituting the government‘s budget constraint $T = \phi R - s_b b - s_R R$ and the profits of the firms into the budget constraint of the individual, we show that the same holds for the capital accumulation equation.
If the social planer values capital, the resource stock and the technology levels in the same way as the subjects of the decentralized economy, $\lambda = \lambda_H$, $\tau = \tau_H$, $\kappa_b = \kappa_{bH}$ and $\kappa_R = \kappa_{RH}$ hold. According to the necessary condition of the individuals optimization problem the marginal utility equals λ_H. An equilibrium on the composite good market requires $\lambda_H = p_Y$, since the individual buys more (less) Y

if $U_C = \lambda_H > \ (<) p_Y$. The equations determining the R&D investments in the decentralized economy establish then the same rules like the equations of the social optimum, if the resource firm with the higher shadow price, i.e. the higher willingness to pay, prevails against the other resource firm on the composite good market. In this case, we get:

- Case (i) is realized, if (I) and (Ia) hold.
- Case (ii) is realized, if (I) and (II) hold.
- Case (iii) is realized, if (I) and (III) hold.
- Case (iv) is realized, if (II) and (III) hold.
- Case (v) is realized, if (V) and $\kappa_{bH} > \kappa_{RH}$ hold.
- Case (vi) is realized, if (VI) and $\kappa_{bH} > \kappa_{RH}$ hold.
- Case (vii) is realized, if (V), (VI) and $\kappa_{bH} = \kappa_{RH}$ hold.

The subsidies adjust for the market power of the resource firms, while the tax covers the pollution effect. Thus, every externality is allocated to one governmental instrument. The terms for the marginal product of the backstop are equal, if $s_b = -p_Y F_{xx} \nu^2 b > 0$. Similarly, the marginal product of the exhaustible resource in the social optimum with $\theta = 0$ is resembled by the decentralized economy with $\phi = 0$, if $s_R = p_Y F_{xx} R > 0$. Pollution implies then $\phi = -\theta > 0$.

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