A note on Nash equilibrium in soccer

Volker Schlepütz*, Hendrik Sonnabend†

FernUniversität Hagen, Department of Economics,

Profilstrasse 8, D-58084 Hagen, Germany

February 10, 2007

Abstract

In this note, we extend the soccer game studied by Moschini (2004) by analysing the sequential game as suggested by him. The analysis supports multiple subgame perfect equilibria. More important, under non-commitment, in equilibrium, no goals are scored at all. This raises the problem that sequential play performes poor in explaining empirical findings on players' strategies in soccer. Reconsidered under an enlarged set of game forms, our results suggest to view soccer most plausibly as a simultaneous move game.

Keywords: Sequential games; Soccer
JEL *classification:* C72

* *Corresponding author*, Tel.: +49 2331 987 2591; fax: +49 2331 987 347. *E-mail address*: volker.schlepuetz@fernuni.hagen.de.

† *E-mail Address:* hendrik.sonnabend@fernuni-hagen.de.

1 Introduction

In a recent contribution, Moschini (2004) brings further statistical support for the notion of mixed-strategy equilibrium using sports data from soccer.[1] In particular, he concentrates on shots on goals taken from off-center positions. For such situations, the conventional wisdom of soccer experts is that the goalkeeper should defend the near post, and the striker should try to score on the far post. To test this hypothesis, Moschini analyses the simultaneous game.

Inspired by the fact that Moschini introduced the game as best be viewed as sequential, in this note, we take the opportunity to analyze the sequential game. The analysis of this game shows that the nature of subgame perfect equilibria crucially depends on the commitment assumption with regard to the goalkeeper's position. Under the most plausible assumption of non-commitment of the goalie's position no goals are scored in equilibrium. This result is explained by the asymmetric nature of the player's Cournot conjectures. Intuitively, since only the striker's shot can most plausibly be considered as a binding commitment, the goalie can match any shot direction having observed the striker's shot. The implication of this result is that the sequential game form is implausible. Thus, soccer should indeed best be viewed as a simultaneous move game.

The paper is organized as follows. In section 2 we restate the main result of Moschini's simultaneous game. Section 3 analyses the sequential game under different commitment assumptions. Section 4 gives an overall conclusion.

2 The simultaneous game

In Moschini's model, the goalkeeper is assumed to choose a position p between the far post (F) and the near post (N). Since the size of the goal is normalized to unity, the goalkeeper's position can be treated as a probability of taking position at the near post. The striker ex-ante is assumed to choose only between a shot directed to the far post or the near post. Viewed as a simultaneous move game, figure 1 shows a simplified version of the normal

[1] Earlier empirical analysis were conducted by Chiappori et al. (2002), and Palacios-Huerta (2003).

		Striker	
		F	**N**
Goal-keeper	**F** 1-p	0	$a_N(\lambda)$
	N p	$a_F(\lambda)$	0

Figure 1: The normal-form representation of the soccer game

form representation of the game analyzed by Moschini.[2]

The payoffs in each cell represent the scoring probabilities of the striker. If the striker directs a shot to the near post, while the goalkeeper takes position at the far post, a goal is scored with probability $a_N(\lambda)\epsilon(0,1)$, where $\lambda\epsilon(0,\overline{\lambda}]$ measures the angle of the striker's position relative to the goal, as well as distance from the goal.[3] Analogous, a shot directed to the far post, while the goalkeeper prepares for a near post shot scores with probability $a_F(\lambda)\epsilon(0,1)$.

By assumption, $a_N(\lambda) > a_F(\lambda)$ for any λ, and $a_i'(\lambda) < 0$ for $i = N, F$. The former means that for any given distance a shot to the near post is more successful in terms of the scoring probability. The latter means that, given a shot direction, the probability of a goal is decreasing in the distance from the goal.

Since the game is zero-sum, the simultaneous game only has a mixed-strategy equilibrium. Denoting q as the striker's probability of choosing N, the equilibrium strategies are given by:[4]

$$p^*(\lambda) = \frac{a_N(\lambda)}{a_N(\lambda) + a_F(\lambda)}, \quad (1)$$

$$q^*(\lambda) = \frac{a_F(\lambda)}{a_N(\lambda) + a_F(\lambda)}. \quad (2)$$

Since $a_N(\lambda) > a_F(\lambda)$, the goalkeeper, in equilibrium, favors the near post

[2] In Moschini's model, the goalkeeper's and the strikers's skills are explicitly modeled. Our simplified version ignores these specifications since our main conclusions are unaffected by such details.

[3] The upper bound on λ can be justified by the boundaries of the playground.

[4] The equilibrium strategies correspond to (3) and (4) in Moschini (2004), p 368.

$(p^* > 1/2)$ while the striker favors the far post $(q^* < 1/2)$.[5] Note that, in equilibrium, expected scoring probability is given by:

$$\pi^*(\lambda) = \frac{a_N(\lambda)a_F(\lambda)}{[a_N(\lambda) + a_F(\lambda)]}. \tag{3}$$

Moschini (2004, p. 367) actually suggested a sequential two-stage game where the goalkeeper selects his position between the far and the near post prior to the shot, and the striker chooses the direction of the shot *having observed* the goalkeeper's position.

While it is reasonable to assume that the probability of a goal scored depends on the distance from the goal, we suggest that distance, *in addition*, might change the *game form*. Beyond a certain distance, due to the ball's increased travel time, the goalie should plausibly be assumed to *react* on a given shot.[6] This argument, in our view, may justify a sequential game.

3 The sequential game

To model the game as sequential, we treat λ as the horizontal distance from the goal for a given off-center position. To fix ideas, let $\widehat{\lambda} < \overline{\lambda}$ be a critical distance. Then, for $\lambda < \widehat{\lambda}$ the ball's travel time is sufficiently low to justify Moschini's simultaneous game, while for further distances, i.e. $\lambda \geq \widehat{\lambda}$, we treat the game as one of sequential moves. Therefore, in the following, $\widehat{\lambda} < \lambda < \overline{\lambda}$ and $a_i(\lambda) > 0$ for $i = N, F$ is assumed.

To take into account the importance of the goalkeeper's commitment of his position and the striker's possibility of mixing between his two actions, we distinguish three cases.

Case 1: Commitment without striker's mixing

Denote $s \epsilon \{N, F\}$ the action set of the striker. The analysis of this game yields the following result.

> **Proposition 1.** If the goalkeeper must commit to his position, the subgame perfect equilibria are characterized by $p^* = a_N(\lambda)/ [a_N(\lambda) + a_F(\lambda)]$ and $s^* = \{N, F\}$.

[5] This corresponds to *Result 1* of Moschini (2004), p 368.

[6] Note that travel time should be considered as the product of the strength of the strikers shot times the distance. For the above argument to hold, we take the strength of the shot as given.

Proof: Given p, the striker's payoff from choosing N in stage 2 is given by $\pi_N(p) = a_N(\lambda)(1-p)$ while choosing F yields the payoff $\pi_F(p) = a_F(\lambda)p$. Thus, the best response correspondence in stage 2 can be derived as:

$$s^*(p) = \left\{ \begin{array}{ccc} N & if & p < \widetilde{p} \\ \{N, F\} & if & p = \widetilde{p} \\ F & if & p > \widetilde{p} \end{array} \right\} \tag{4}$$

where $\widetilde{p} := a_N(\lambda) / [a_N(\lambda) + a_F(\lambda)]$.

In stage 1, the goalkeeper chooses a strategy that minimizes the striker's scoring probability, anticipating that the striker behaves according to his best response correspondence given in (4). As long as $p < \widetilde{p}$ holds, the striker's best response is to choose $s = N$. The scoring probability in the relevant interval is then given by $\pi_N(p) = a_N(\lambda)(1-p)$. This payoff is minimized for the *highest* possible value of p within $[0, \widetilde{p})$. Denote this value as $p_h(N) = \widetilde{p} - \epsilon$, and $\epsilon > 0$. If the goalkeeper chooses $p > \widetilde{p}$, the relevant scoring probability becomes $\pi_F(p) = a_F(\lambda)p$, which is minimized for the *lowest* possible value of p within $(\widetilde{p}, 1)$. Denote this value as $p_l(F) = \widetilde{p} + \epsilon$. For $p = \widetilde{p}$, the striker's best response is either $s = N$ or $s = F$. Note that $\pi_N(\widetilde{p}) = \pi_F(\widetilde{p})$ since the striker is indifferent between his pure strategies. Since

$$\pi_N(\widetilde{p} - \epsilon) > \pi_N(\widetilde{p}) = \frac{a_N(\lambda)a_F(\lambda)}{[a_N(\lambda) + a_F(\lambda)]} = \pi_F(\widetilde{p}) < \pi_F(\widetilde{p} + \epsilon) \tag{5}$$

holds, it follows that $p^* = \widetilde{p}$ is the global minimizer among $p \epsilon [0, 1]$. $\square$

Proposition 1 states that there are two subgame perfect equilibria where the striker shoots to one post *for certain.* This implies that equilibrium play in the sequential game, contrary to the simultaneous game, is not unique. Moreover, as can be easily verified, the scoring probability in either equilibrium is equal to the expected scoring probability of the mixed-strategy equilibrium in the simultaneous move game. However, the scoring probability in any sequential equilibrium is *decreased* relative to the expected one in the simultaneous game. This is explained by the further distance.[7] It implies

[7] To see this, observe from (3) that:

$$\frac{\partial \pi(\lambda)}{\partial \lambda} = \frac{a'_F(\lambda)\, [a_N]^2 + a'_N(\lambda)\, [a_F]^2}{[a_N(\lambda) + a_F(\lambda)]^2}$$

This expression is negative since $a'_i(\lambda) < 0$ and $a_i(\lambda) > 0$ for $i = F, N$ by assumption.

that if the striker could choose the position from where the shot originates, he has an incentive to move the ball forward to the goal into a region where the game is one of simultaneous moves. The sequential move game, thus, seems to be implausible from the striker's view.

Case 2: Commitment with striker's mixing

In Moschini's model, mixing is required for the existence of a mixed equilibrium. In the sequential game, it is not. Yet, allowing the striker to mix yields our next result.

> **Proposition 2.** If the striker is assumed to mix between the far and the near post, the subgame perfect equilibria are characterized by $p^* = a_N(\lambda)/\left[a_N(\lambda) + a_F(\lambda)\right]$, and $q^* = [0, 1]$.

Proof: Given p, the striker's expected scoring probability from mixing in stage 2 is given by:

$$E\pi = qa_N(\lambda)(1-p) + (1-q)a_F(\lambda)p. \tag{6}$$

Note that expected payoff is linear in q, and that marginal expected payoff is given by:

$$\frac{\partial E\pi}{\partial q} = (1-p)a_N(\lambda) - pa_F(\lambda). \tag{7}$$

Marginal expected payoff is increasing in p for $p < \widetilde{p}$. Thus, $q^* = 1$ for $p\epsilon[0, \widetilde{p})$. Marginal expected payoff is decreasing in p for $p > \widetilde{p}$. Thus, $q^* = 0$ for $p\epsilon(\widetilde{p}, 1]$. Finally, marginal expected payoff is constant for $p = \widetilde{p}$. Hence, the only difference to the best response correspondence in (3) is that the striker in the case of $p = \widetilde{p}$ is indifferent between N and F, and *all mixed strategies* between his two actions. Observe that the goalie's expected scoring probability is independent of q, and equal to $\pi_N(\widetilde{p}) = \pi_F(\widetilde{p})$. Hence, the optimal position of the goalie in stage 1 is unaffected by the striker's mixing, and follows from the proof of proposition 1. However, the range of equilibrium strategies enlarges to infinity. □

Proposition 2 implies that under the striker's possibility of mixing the mixed-strategy equilibrium of the simultaneous game is one out of infinitely many equilibria of the sequential game. Thus, equilibrium play of the sequential game does not *uniquely* explain the empirical findings. Indeed, *any* distribution of shots on the goal were consistent with equilibrium play of the sequential game.

Case 3: The No-Commitment game

If the commitment assumption of the goalkeeper is dropped, it is possible for him to correct his first-stage position after the striker has chosen his shot direction. As a consequence, the striker should ignore the first stage. Therefore, in the remaining subgame, the striker becomes the leader who first strikes a shot, and the goalie, thereafter, chooses his position having observed the shot. The key point is that, unlike the goalkeeper's position, the striker's shot naturally is a binding commitment. Therefore, the standard argument of Fudenberg and Tirole (1991, p. 77) that the Stackelberg game without binding commitment yields the simultaneous equilibrium fails to hold in our game since Cournot conjectures are *asymmetric* between the two players. Moreover, being the leader in the game renders the striker's mixing meaningless since he cannot make the goalie guessing. From these observations, our final proposition follows straightforward, and without formal proof.

> **Proposition 3.** The no-commitment game has two subgame perfect equilibria, $(s^* = N, p^* = 1)$ and $(s^* = F; p^* = 0)$.

Most important, proposition 3 implies that no goals are scored in equilibrium, since in either equilibrium the scoring probability is zero. This is explained by the fact that the goalie keeps any shot. This result is in stark contrast to the prediction of mixed-equilibrium play where goals are scored with positive probability. Thus, the analysis of the most plausible game form yields a rather implausible result viewed from real play since even far-shots sometimes result in goals.

4 Conclusion

Economists feel highly uncomfortable in the case their predictions of theoretical models are not supported by empirical findings. This is true also for the study of soccer games. The analysis of soccer as a sequential game increases the discrepancy between game theoretical predictions and empirical findings even more. Most important, goals at all cannot be explained by data under the most plausible assumption of non-commitment by the goalie. This leads us to conclude that soccer should *not* be treated as a sequential game, as suggested by Moschini. It is indeed best be viewed as a simultaneous move game.

References

Chiappori, P.A., Levitt, S., Groseclose, T., 2002. Testing mixed strategy equilibria when players are heterogeneous: the case of penalty kicks in soccer. American Economic Review 92 (4), 1138-1151. (September).

Fudenberg, D. and J. Tirole, 1991, Game Theory. (MIT, Cambridge).

Moschini, G.C., 2004, Nash equilibrium in strictly competitive games: live play in soccer. Economics Letters 85 (3), 365-371. (December).

Palacios-Huerta, I., 2003. Professionals play minimax. Review of Economic Studies 70 (2), 395-415.