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Volker Schlepütz

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Consulting incentives in contests

Volker Schlepütz*
FernUniversität in Hagen, Department of Economics,
Profilstrasse 8, D-58084 Hagen, Germany

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Abstract

We study an asymmetric contest where a favorite has the option to consult underdogs instead of taking part in the contest as a player. As a consultant, a favorite does not take part in the contest but communicates his superior knowlegde to underdogs in exchange for being compensated by them by a share of the prize they win in the smaller contest as consulted players. It is shown that the favorite has consulting incentives if the number of underdogs is sufficiently high. Moreover, consulting avoids overdissipation or rents.

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*corresponding address: volker.schlepuetz@fernuni-hagen.de; phone: +49(0)2331 987 2591, fax: +49(0)2331 987 347.

1 Introduction

The existence of consulting firms is well understood and documented in business economics. An excellent overview gives Armbrüster (2006). In economics, especially in industrial organization, incentives to act as a consultant are far less investigated. Even in the literature on strategic information transmission in the tradition of Crawford and Sobel (1982) the focus is more on the efficient use of information than on the incentives to act as an expert at all.[1] An explanation of the pure *existence* of consulting firms is, thus, still missing. The fact that consulting exists in many markets raises the question of *why* it exists. In this paper, we investigate incentives to act as a consultant in contests and study the welfare effects of a possible consulting equilibrium compared to the absence of consulting as an outside option.

In symmetric contests consulting is senseless since no individual has superior knowledge which could be communicated to other individuals. However, in asymmetric contests, players have different abilities. Favorites compete with underdogs for winning a prize of a given size. To bring the idea of consulting into question, we define a consulted contest as follows: A consulted contest is a contest where the favorite drops out of the contest with the possibility to transfer his superior information to underdogs.

Our paper is mostly related to the literature on entry to and exit from contests in line of Corcoran (1984), and more recently advanced by Matros (2006) and Morgan et al. (2008). The common inside from this literature is that, under free entry, the entry of players occurs until positive expected payoffs are dissipated.

Matros (2006) investigates the comparative static properties of the addition or the deletion of a player in an asymmetric contest. However, he neither investigates whether the deletion of a player is an optimal choice of the player, nor does he analyze the effects on overall rent-seeking activities in the contest.

Morgan et al. (2008) considers the endogenous choice of opting for the contest as an alternative to an exogenously given outside option. Contrary to Morgan (2008), the payoff of the outside option in our paper is not assumed to be taken exogenously. Rather, the consulting payoff is determined endogenously. With an exogenously assumed payoff as an outside option, the explanation for purely opting out seems to be rather ad hoc, because opting

[1] See Valsecchi (2008) for a recent review of this literature.

out can always be defined as a result of access to some proper defined wealth. We try to explain consulting incentives by comparing payoffs which can *only* be obtained in competition for rents without access to an exogenously given wealth.

Our paper is also related to the literature on delegation contests first investigated by Baik and Kim (1997). In delegation contests, a main player can decide to act in the contest himself or to let a delegate with a different valuation of the price act in the contest on behalf of him. The idea of consulting is different from delegation. First, delegates may have a superior ability compared to the main player. A consulted player, however, can never be more able than the player he is consulted from. More important, whether the main player or the delegate acts in the contest, the actual number of players in delegated contest is unchanged. Contrary to this, a consulted contest is reduced by the number of consultants. We, therefore, do not explain why competition for rents is partly or completely *substituted* by superior players. Instead, we explain, why superior players have an incentive to voluntarily *drop out* of the competition for rents and instead consult the remaining contestants.

The remainder of the paper is organized as follows. Section 2 derives consulting incentives in a two-stage game and investigates the effect of a possible consulting equilibrium on total rent-seeking efforts. Concluding remarks are given in Section 3.

2 The model

Consider a population of $n + 1$ risk neutral individuals intending to engage in a contest to win a prize with value V. Among the population of players one player is assumed to be a favorite valuing the prize with its true value. The remaining n players are assumed to underestimate the true value such that they value the prize at θV where $0 < \theta < 1$. Thus, these players are underdogs due to a biased valuation of the prize.[2] The favorite is assumed to be able to consult underdogs. Consulting communicates the true value of the prize to the underdogs. We are interested in the favorite's incentives

[2] We have investigated incentives to consult underdogs under the alternative assumption of different abilities. The analysis shows that consulting incentives can be derived by the solution of a cubic equation. To make the model most tractable for the purpose of explaining consulting incentives the main inside of the paper is unchanged by the simplified version of modelling asymmetry by a biased valuation of the prize.

to act as a consultant and in the welfare effects of consulting in the case it occurs.

In line with the rent-seeking contest of Tullock (1980), we apply the usual probability of winning the prize given by:

$$p_i = \frac{e_i}{e_i + \sum_{j \neq i}^{n} e_j} \text{ for } i = 0, ..., n \tag{1}$$

where $i = 0$ indicates the favorite's identity. As a consultant, the favorite is assumed to be able to perfectly communicate the true valuation of the price to all players willing to be consulted ones without costs. Costless consulting can be rationalized by the fact that the favorite already knows the true value of the prize and the costs of communication is neglectable.

2.1 The consulting game

We consider the following game. The favorite is supposed to first announce whether she will expend her effort in a contest with the underdogs, or consult a subset of underdogs without taking part in the contest. If the favorite announces to act as a player, the second stage of the game is an asymmetric contest. Otherwise, she announces a consulting fee α, i.e. a share of the prize as her compensation for consulting. Thus, we adopt a simplified version of the payoff structure used in delegation contests:

$$C_0 = \left\{ \begin{array}{ll} \alpha V & \text{if a consulted player } i \text{ wins the prize} \\ 0 & \text{otherwise.} \end{array} \right\} \tag{2}$$

Given a consulting offer, stage 2 of the game is reached. In this subgame, underdogs are assumed to accept or reject a consulting offer. If an underdog rejects an offer he remains an underdog in the subsequent contest. Otherwise, he enters the subsequent contest as a favorite and, in the case he wins, shares the prize with the consultant. As usual, we employ a subgame-perfect equilibrium as the solution concept.

2.1.1 The no-consulting subgame

In the no-consulting subgame, the favorite and the underdogs compete with each other in the contest. Using (1), the expected payoff EU_0 for the favorite and EU_i for each underdog are $U_0 = Vp_0 - e_0$ and $U_i = V\theta p_i - e_i$. This subgame has a unique Nash equilibrium in pure strategies, where all players take part in the contest. We obtain the following lemma.

Lemma 1. At the Nash equilibrium of the no-consulting subgame, the equilibrium effort levels and the expected payoffs of the favorite and the underdogs are given by

$$e_0^* = Vn\left[\frac{\theta + n(1-\theta)}{\theta + n}\right] \quad (3)$$

$$e_i^* = Vn\left[\frac{\theta}{\theta + n}\right] \text{ for } i = 1, ..., n \quad (4)$$

$$EU_0^* = V\left[\frac{\theta + n(1-\theta)}{\theta + n}\right]^2 \quad (5)$$

$$EU_i^* = V\theta\left[\frac{\theta}{\theta + n}\right]^2 \text{ for } i = 1, ..., n. \quad (6)$$

Most important, (5) defines the *reservation utility* of the potential consultant. Note also that the sum of rent seeking efforts amounts to $e_0^* + ne_i^* = Vn$. Thus, in our asymmetric contest overdissipation of rents occurs. It is therefore interesting to investigate, whether a consulted contest avoids this overdissipation of rents.

2.1.2 The consulting subgame

In the consulting subgame, each player is assumed to choose whether he wants to be a consulted player, or remains an underdog. Denote $\delta_i \epsilon \{0, 1\}$ each player's demand for consulting where $\delta_i = 1$ represents the decision to enter the contest as a consulted player. The analysis of this games allows us to state the following lemma.

Lemma 2. If and only if $\alpha \leq 1 - \theta$, then $\delta_i^* = 1$ and $e_i^* = (1-\alpha)V(n-1)/n^2$ is a Nash equilibrium best response of all players in the $n-$player contest.

Proof: Define $v := (1-\alpha)$ and $w := \theta$. If all players decide to become consulted ones in the contest, optimal effort of each player is derived from the utility maximization problem:

$$\max_{e_i} vVp_i - e_i. \quad (7)$$

From the first order conditions, we obtain the optimal effort levels:

$$e_i^{**} = vV(n-1)/n^2 \quad (8)$$

generating an equilibrium utility of:

$$EU_i^{**} = vV/n^2. \tag{9}$$

To see that this is an equilibrium best response to $\alpha \leq 1-\theta$, let us denote $e^*_{i\neq k}$ as the effort level of all other players except player k. Setting $\delta_k = 0$, the optimal effort adjustment of player k given the other players equilibrium efforts from (8) amounts to:

$$\widehat{e}_k = \frac{V(n-1)}{n^2}\left[n\sqrt{vw} - v(n-1)\right]. \tag{10}$$

Note that the underdog may be inactive in the contest. In this case, his utility is zero, and the decision of $\delta_k = 0$ is dominated by $\delta_k^* = 1$. As an active player, inserting the optimal deviation effort in (10) into his underdog's utility, we get:

$$\widehat{EU}_k = \frac{V}{n^2}\left[n\sqrt{vw} - v(n-1)\right]\left[\frac{wn - \sqrt{vw}}{\sqrt{vw}}\right]. \tag{11}$$

Compare (9) and (11) to see that

$$EU_i^* \lesseqgtr \widehat{EU}_k \Longleftrightarrow v \gtreqless w.$$

From the last relation, lemma 2 follows. Q.E.D.

The intuition behind lemma 2 is easily explained by the fact that players only demand consulting if the subsequent contest leaves them at least indifferent between consulting and acting as an underdog. This inside follows from the general nature of demand in the sense that the price for a good should not exceed the willingness to pay in order to consume it. In our case, willingness to pay is the equilibrium utility which can be gained as an underdog. As long as the equilibrium utility from acting as a consulted player is at least as high as that possible by an underdog a consulting demand arises. This is equivalent to say that a consulting fee α must not be too high as to generate this demand.

2.2 Consulting incentives

The analysis of the subgames allows us to derive consulting incentives by the favorite. In stage 1, the potential consultant is faced with the following maximization problem:

$$\delta_0^* = \arg\max_{\delta_0 \epsilon \{0,1\}}\left\{EU_0^*, \max_{\alpha,\text{ s.t. } \alpha\leq 1-\theta}\frac{x}{n}C_0\right\}, \tag{12}$$

where $\delta_0 = 1$ denotes the choice of acting as a consultant and earning an expected prize of $\frac{x}{n}C_0(\alpha)$ with x/n being the probability that the potential consultant gets a share of the prize if x players demand consulting, and $\delta_0 = 0$ denotes the decision to act as a player in the asymmetric contest. Since $x = n$ for $\alpha \leq 1 - \theta$ from lemma 2, expected consulting earnings for a consultant are maximized for $\alpha^* = 1 - \theta$. Since the consultant is a monopolist, he is able to completely skim up the willingness to pay for consulting of the players. Using lemma 1, consulting incentives arise if

$$(1-\theta)V \geq V\left[\frac{\theta + n(1-\theta)}{\theta + n}\right]^2 \tag{13}$$

holds. Algebraic manipulations of (13) yield our first proposition.

Proposition 1. If $n \geq \frac{\theta}{\sqrt{1-\theta}}$, consulting occurs in a subgame-perfect equilibrium of the game.

Proposition 1 shows that consulting incentives arise for a broad range of parameter values, especially for a sufficiently high number of underdogs. Only in small number contest in combination with a low biased valuation of the prize by underdogs consulting cannot be explained.

More important, consulting is completely effectless with regard to the subsequent contest. Consulted players behave as if they were underdogs. This is explained by the fact that under equilibrium payment players are indifferent between entering the contest as a favorite, and entering as an underdog.

Our last step is to analyze whether consulting is a means to reduce wasteful rent-seeking activities. This analysis is summarized in our last proposition:

Proposition 2. Consulting always avoids overdissipation of rents.

Proof: From (8) we get

$$ne_i^{**} = V\theta\frac{(n-1)}{n} < V < Vn = e_0^* + ne_i^*. \tag{14}$$

Q.E.D.

Proposition 2 makes clear that consulting is a means to reduce wasteful rent-seeking activities. This is an important message since consulting has thus far not been investigated in contests as a mean to reduce wasteful rent-seeking. Consulting, thus, is efficient subject to the incentive compatibility constraint the favorite is faced with.

3 Conclusion

We have investigated the incentives of acting as a consultant for underdogs by a favorite instead of his taking part in the contest as a contestant. As has been shown, consulting incentives arise for a sufficiently high number of contestants. Thus, our analysis explains the existence of consulting firms in contests. Moreover, consulting *should* occur since it avoids the overdissipation of rents otherwise happening in a contest without the possibility of consulting as an outside option for more able players. Consulting should therefore be supported by a contest designer.

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