



**FernUniversität in Hagen**

**SEM modeling with  
singular moment matrices**

**Part II: ML-Estimation of sampled stochastic  
differential equations**

Hermann Singer

Diskussionsbeitrag Nr. 442  
August 2009

Diskussionsbeiträge der Fakultät für Wirtschaftswissenschaft  
der FernUniversität in Hagen

Herausgegeben vom Dekan der Fakultät

Alle Rechte liegen bei den Autoren

# SEM modeling with singular moment matrices

## Part II: ML-Estimation of sampled stochastic differential equations

Hermann Singer  
FernUniversität in Hagen \*

August 27, 2009

### Abstract

Linear stochastic differential equations (SDE) are expressed as an exact discrete model (EDM) and estimated with structural equation models (SEM) and the Kalman filter (KF) algorithm. The SEM likelihood is well defined even for the times series case and the SEM and KF approach yield the same likelihood. The oversampling approach is introduced in order to formulate the EDM on a time grid which is finer than the sampling intervals. This leads to a simple computation of the nonlinear parameter functionals of the EDM. For small discretization intervals, the functionals can be linearized and software permitting only linear parameter restrictions can be used. However, in this case the SEM approach must handle large matrices leading to degraded performance and possible numerical problems. The methods are compared using coupled linear random oscillators with time varying parameters and irregular sampling times.

**Key Words:** Structural Equation Models (SEM); Kalman filtering (KF); Stochastic differential equations (SDE); Maximum Likelihood (ML) estimation; Nonlinear parameter restrictions; Oversampling; Coupled stochastic oscillators

## 1 Introduction

In a companion paper (I, Singer; 2009) it was shown that the structural equation model (SEM) representation of a discrete time state space model yields

---

\*Lehrstuhl für angewandte Statistik und Methoden der empirischen Sozialforschung, D-58084 Hagen, Germany, hermann.singer@fernuni-hagen.de

correct ML estimates even for one panel unit ( $N = 1$ ), when the joint Gaussian distribution of the observations is used as likelihood function.

In this paper, the representation of sampled continuous time stochastic processes in terms of the exact discrete model (EDM; Bergstrom; 1976b, 1988) is discussed. This vector autoregression (VAR) involves nonlinear matrix restrictions for the parameter matrices, which must be handled by the software. A Kalman filter approach was given by Singer (1991, 1993, 1995, 1998). Nonlinear (w.r.t. the parameters) SEM software can implement such restrictions also (Oud and Jansen; 2000), but it may be worthwhile to find a representation which can be used in linear SEM and time series programs.

I propose the device of *oversampling* in order to linearize the EDM at a latent discretization level which is smaller than the sampling interval (Singer; 1995). The price to be paid is a larger dimension of the latent state, but now the parameter matrices are approximately linear with arbitrarily small error and the parameter functionals w.r.t. the exogenous variables are computed automatically by the SEM equations. The enlargement of the latent state dimension is unproblematic for the recursive Kalman approach, however (Singer; 1995).

Furthermore, oversampling yields a simple treatment of irregular sampling, since only the parameter matrices of the latent VAR recursion are involved. Time dependent parameters and arbitrary interpolation schemes of the exogenous variables are easily implemented in this approach.

This will be detailed in the further sections. Section 2 gives the definition of a SEM model including deterministic intercept terms and states the Gaussian likelihood function. In section 3 the SEM representation is applied to sampled stochastic differential equations (SDE). Using oversampling, linearized parameters with small errors are obtained, but the nonlinear restrictions are preserved. In section 4, the method is applied to coupled stochastic oscillators. In two appendices, oversampling and computational aspects are discussed.

## 2 SEM modeling

In the following the SEM model

$$\eta_n = B\eta_n + \Gamma x_n + \zeta_n \quad (1)$$

$$y_n = \Lambda\eta_n + \tau x_n + \epsilon_n \quad (2)$$

$n = 1, \dots, N$ , will be considered. The structural matrices have dimensions  $B : P \times P$ ,  $\Gamma : P \times Q$ ,  $\Lambda : K \times P$ ,  $\tau : K \times Q$  and  $\zeta_n \sim N(0, \Sigma_\zeta)$ ,  $\epsilon_n \sim N(0, \Sigma_\epsilon)$  are independent normally distributed error terms ( $\Sigma_\zeta : P \times P$ ,  $\Sigma_\epsilon : K \times K$ ).

In the structural and the measurement model, the variables  $x_n$  are *deterministic* control variables. They can be used to model intercepts and for dummy coding. Stochastic exogenous variables  $\xi_n$  are included by extending the latent

variables  $\eta_n$ . Since the error vectors are normally distributed, the indicators (2) are distributed as  $N(\mu_{y_n}, \Sigma_y)$ , where

$$\eta_n = B_1(\Gamma x_n + \zeta_n) \quad (3)$$

$$E[\eta_n] = B_1\Gamma x_n \quad (4)$$

$$\text{Var}(\eta_n) = B_1\Sigma_\zeta B_1' \quad (5)$$

$$E[y_n] = \mu_{y_n} = \Lambda E[\eta_n] + \tau x_n = [\Lambda B_1\Gamma + \tau]x_n := Cx_n \quad (6)$$

$$\text{Var}[y_n] = \Sigma_y = \Lambda \text{Var}(\eta_n)\Lambda' + \Sigma_\epsilon = \Lambda B_1\Sigma_\zeta B_1'\Lambda' + \Sigma_\epsilon. \quad (7)$$

In the equations above, it is assumed that  $B_1 := (I - B)^{-1}$  exists. Thus, the log likelihood function for the  $N$  observations  $\{y_n, x_n\}$  is

$$l = -\frac{N}{2} \left( \log |\Sigma_y| + \text{tr}[\Sigma_y^{-1} \frac{1}{N} \sum_n (y_n - \mu_{y_n})(y_n - \mu_{y_n})'] \right). \quad (8)$$

In order to implement arbitrary restrictions on the structural matrices, it is assumed that they depend on an  $u$  dimensional parameter vector  $\psi$ , e.g.  $\Sigma_\zeta = \Sigma_\zeta(\psi)$  etc. For example, the matrix exponential function in the definition of the exact discrete model (section 3) must be specified.

The likelihood function (8) is well defined for  $N = 1$ , since no log determinants of the moment matrices are involved, as is suggested by the ML fitting function of LISREL (cf. LISREL 8 reference guide, p. 21, eqns. 1.14, 1.15, p. 298, eqn. 10.8). The covariance matrix of the indicators,  $\Sigma_y$  (eqn. 7), must be nonsingular, however.<sup>1</sup> For a more detailed discussion of the likelihood function, see I.

## 3 Stochastic differential equations and continuous-discrete state space models

### 3.1 Stochastic differential equations

The stochastic differential equation (SDE)

$$dy(t) = A(t)y(t)dt + b(t)dt + G(t)dW(t); t \in [t_0, t] \quad (9)$$

is a linear dynamical model for the time evolution of the continuous time state vector  $y(t) : p \times 1$ . The matrix  $A(t) : p \times p$  is called the drift,  $b(t)$  is an inhomogenous term and  $G(t) : p \times r$  is the strength of the process error  $dW(t) : r \times 1$  where  $W(t)$  is the  $r$ -dimensional Wiener process (Arnold; 1974). Formally one can divide by  $dt$  to obtain the white noise process  $\zeta(t) = dW(t)/dt$ , but  $W(t)$  (a continuous time random walk) is not differentiable.

<sup>1</sup>Otherwise the singular normal distribution can be used (Mardia et al.; 1979, p. 41).

Models of this kind have a long tradition of applications in many fields, such as physics, mathematics, engineering, econometrics, finance, sociology, psychology etc.<sup>2</sup>

In applications, the inhomogeneous term  $b(t)$  is usually parameterized as

$$b(t) = B(t)x(t); B(t) : p \times q$$

where  $x(t)$  are deterministic control variables. In the panel case, one writes  $b_n(t) = B(t)x_n(t)$ . The parameter matrices are assumed to depend on a parameter vector  $\psi$ , which is suppressed in most formulas.

The solution of (9) with initial value  $y(t_0)$  is given by

$$y(t) = \Phi(t, t_0)[y(t_0) + \int_{t_0}^t \Phi(s, t_0)^{-1}[b(s)ds + G(s)dW(s)], \quad (10)$$

(Arnold; 1974, ch. 8) where  $\Phi(t, t_0)$  is the fundamental matrix of the system solving the deterministic matrix differential equation

$$\dot{\Phi}(t, t_0) = A(t)\Phi(t, t_0); \Phi(t_0, t_0) = I. \quad (11)$$

It can be solved by a time ordered matrix exponential or as (time ordered) infinite product

$$\Phi(t, t_0) = \overleftarrow{T} \exp\left[\int_{t_0}^t A(s)ds\right] \quad (12)$$

$$= \overleftarrow{T} \lim_{J \rightarrow \infty} \prod_{j=0}^{J-1} [I + A(\tau_j)\delta t], \quad (13)$$

$\tau_j = t_0 + j\delta t; \delta t = (t - t_0)/J$ , where  $\overleftarrow{T} [A(t)A(s)] = A(s)A(t); t < s$  is the Wick time ordering operator (cf. Abrikosov et al.; 1963). The time ordering is essential, since the matrices  $A(t)$  do not commute in general, i.e.  $A(t)A(s) \neq A(s)A(t)$ .

### 3.2 Exact discrete model (EDM)

The solution of the SDE (10) is the basis of the *exact discrete model* (EDM) introduced by (Bergstrom; 1976a, 1988), valid at the time points of measurement

---

<sup>2</sup>cf. e.g., Langevin (1908); Uhlenbeck and Ornstein (1954); Bartlett (1946, 1955, 1978); Itô (1951); Stratonovich (1960); Kalman and Bucy (1961); Schweppe (1965); Nelson (1967); Bergstrom (1976a); Coleman (1968); Jazwinski (1970); Black and Scholes (1973); Jennrich and Bright (1976); Phillips (1976); Doreian and Hummon (1976, 1979); Jones (1984); Möbus and Nagl (1983); Arminger (1986); Singer (1986, 1990, 1992b, 1998); Zdrozny (1988); Hamerle et al. (1991, 1993); Oud and Jansen (2000); Oud and Singer (2008a,b).

$\{t_0, t_1, \dots, t_T\}$ ,

$$\begin{aligned} y(t_{i+1}) &= \Phi(t_{i+1}, t_i)y(t_i) \\ &+ \int_{t_i}^{t_{i+1}} \Phi(t_{i+1}, s)b(s)ds \\ &+ \int_{t_i}^{t_{i+1}} \Phi(t_{i+1}, s)G(s)dW(s), \end{aligned}$$

which is abbreviated as the VAR(1) system

$$y_{i+1} = A_i^*y_i + b_i^* + u_i; \quad i = 0, \dots, T-1. \quad (14)$$

The EDM is a Gaussian vector autoregression, but with nonlinear parameter restrictions involving the structural matrix  $A(t)$  and the sampling intervals  $t_i$ . It is of the same form as the discrete time dynamical system (I, eq. 9-10).

An important special case is the system with constant coefficients  $A(t) = A, b(t) = Bx(t), G(t) = G; \Omega = GG'$ . In this case the parameter functionals are given explicitly as

$$\Phi(t_{i+1}, t_i) = A_i^* = \exp[A(t_{i+1} - t_i)] := \exp(A\Delta t_i) \quad (15)$$

$$b_i^* = \int_{t_i}^{t_{i+1}} \exp[A(t_{i+1} - s)]Bx(s)ds \quad (16)$$

$$\begin{aligned} \Omega_i^* &:= \text{Var}(u_i) \\ &= \int_{t_i}^{t_{i+1}} \exp[A(t_{i+1} - s)]\Omega \exp[A'(t_{i+1} - s)]ds \end{aligned} \quad (17)$$

$$\text{row}(\Omega_i^*) = [A \otimes I + I \otimes A]^{-1}[A_i^* \otimes A_i^* - I] \text{row}(\Omega) \quad (18)$$

where  $\text{row}$  is the row-wise vector operator and  $\otimes$  is the Kronecker product (McDonald and Swaminathan; 1973).

The estimation of the EDM (14) requires the implementation of the correct parameter restrictions, most importantly the matrix exponential function

$$\exp(A\Delta t_i) = \sum_{l=0}^{\infty} (A\Delta t_i)^l / l! \quad (19)$$

$$= \prod_{j=0}^{J_i-1} \exp(A\Delta t_i / J_i) \quad (20)$$

$$= \lim_{J_i \rightarrow \infty} \prod_{j=0}^{J_i-1} [I + A \cdot (\Delta t_i / J_i)]. \quad (21)$$

It contains the parameters  $a_{mn}; m, n = 1, \dots, p$  in a complicated nonlinear manner and is dependent on the sampling intervals  $\Delta t_i$ . Thus, even for constant dynamics  $A$ , the measurements depend on the sampling scheme. The product representation (21) shows, that the nonlinear transition matrix may be viewed as

the product of linear (in  $A$ ) transition matrices  $I + A \cdot (\Delta t_i / J_i) = I + A \delta t$  defined on smaller discretization intervals  $\delta t = \Delta t_i / J_i$  (Singer; 1995, 1998). This introduction of a smaller discretization interval is called *oversampling*. It may be achieved by the definition of additional latent variables  $y(t)$ , where  $t$  is between the measurement times  $\{t_0, t_1, \dots, t_T\}$ .

Summarizing the discussion,

- the crucial point is the computation of the parameter functionals (15–17). This point relates to any approach, either Kalman filter or SEM representations. In LSDE (Singer; 1990, 1991), the matrix exponential and its derivative was computed using the spectral decomposition of  $A$  (cf. also Jennrich and Bright; 1976; Jones; 1993; Najfeld and Havel; 1995; Moler and VanLoan; 2003).
- The influence of the exogenous variables  $b(s) = Bx(s)$  is present during the whole sampling interval  $[t_i, t_{i+1}]$  (see 16). Since the data  $x(s)$  are usually only available at some time points  $t_i$ , approximations of  $x(s)$  must be used, such as step functions, polygons or other interpolation schemes (cf. Phillips (1976); Singer (1995)). Alternatively, one can use stochastic variables (by extending the state vector) which are automatically interpolated by computing the smoothed states  $E[x(t)|z_T, \dots, z_0]$ . For example, using the trivial dynamics  $dx = GdW$ , polygonal lines are obtained.
- The integral (16) must be solved explicitly to implement the restrictions. In the oversampling approach, these computations are done implicitly by the system equations.

### 3.3 State space modelling

Usually the states  $y(t_i)$  are not directly observable, therefore a measurement model

$$z_i = H(t_i)y(t_i) + d(t_i) + \epsilon_i; i = 0, \dots, T,$$

$\text{Var}(\epsilon_i) = R_i$  has to be added to the dynamic system model. Together with (9) one speaks of the *continuous-discrete state space model*

$$dy(t) = A(t)y(t)dt + b(t)dt + G(t)dW(t); t \in [t_0, t_T] \quad (22)$$

$$z_i = H_i y_i + D_i x_i + \epsilon_i, i = 0, \dots, T. \quad (23)$$

As in the dynamical model, exogeneous variables  $x(t) : q \times 1$  are used to specify the intercept terms  $d(t) = D(t)x(t)$ . In the panel case, one writes  $d_n(t_i) = D_i x_{ni}$ . All matrices depend on an  $u$ -dimensional parameter vector  $\psi$  and, if necessary, on lagged measurements  $Z^{i-1} = \{z_{i-1}, \dots, z_0\}$ .

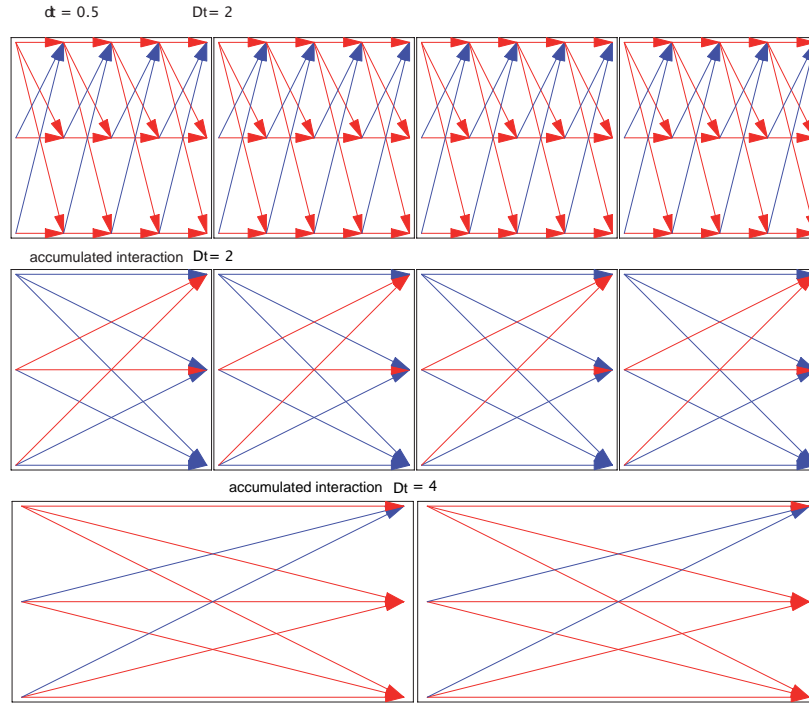


Figure 1: Oversampling of measurement intervals  $\Delta t = 2$  and  $\Delta t = 4$  with discretization interval  $\delta t = 0.5$ . The discrete time parameters  $A^*(\Delta t) = \exp(A\Delta t)$  have different signs (red = +, blue = -) although the continuous time drift  $A$  is constant.

### 3.3.1 Oversampling

The dynamical EDM (14) is usually formulated for the measurement times  $t_i$ . Then, the computation of the parameter functionals in (14) requires the solution of the fundamental matrix and integrals of  $x(s)$  over the sampling intervals  $\Delta t_i$ . To obtain an explicit solution, the exogenous variables must be interpolated between  $x(t_i)$  and  $x(t_{i+1})$ . As mentioned, usually step or polygonal approximations are used. For the fundamental matrix  $\Phi$ , one can use time ordered matrix exponentials (Singer; 1998) or a product representation (cf. eqn. 12–13). In the oversampling approach (fig. 1), arbitrary interpolation schemes for  $x(t)$  such as splines etc. can be used easily, since the integrals are obtained automatically by using a finer discretization scheme. The EDM is formulated at time points  $\tau_j = t_0 + j\delta t, j = 0, \dots, J$ , such that all measurement times can be expressed on this grid with uniform spacing  $\delta t$ , i.e.  $t_i = \tau_{j_i}$  with indices  $j_0 = 0, \dots, j_T = J = (t_T - t_0)/\delta t$ .

This approach (cf. Singer; 1995, 2007) may be called *oversampling*, since more



latent states are inserted between the measurement times  $t_i$ . Thus one obtains

$$y_{j+1} = A_j^* y_j + b_j^* + u_j; j = 0, \dots, J \quad (24)$$

$$z_i = H_{j_i} y_{j_i} + D_{j_i} x_{j_i} + \epsilon_{j_i}; i = 0, \dots, T, \quad (25)$$

where

$$A_j^* = \Phi(\tau_j + \delta t, \tau_j) \quad (26)$$

$$b_j^* = \int_{\tau_j}^{\tau_j + \delta t} \Phi(\tau_j + \delta t, s) B(s) x(s) ds \quad (27)$$

$$\Omega_j^* := \text{Var}(u_j) = \int_{\tau_j}^{\tau_j + \delta t} \Phi(\tau_j + \delta t, s) \Omega(s) \Phi'(\tau_j + \delta t, s) ds \quad (28)$$

In contrast to the usual EDM valid at fixed measurement times  $t_i$ , the interval  $\delta t$  can be made arbitrarily small. Then one obtains approximately by linearization<sup>3</sup>

$$A_j^* \approx I + A_j \delta t \quad (29)$$

$$b_j^* \approx b_j \delta t = B_j \hat{x}_j \delta t \quad (30)$$

$$\Omega_j^* \approx \Omega_j \delta t \quad (31)$$

and thus finds the Euler-Maruyama approximation of the SDE on the  $\delta t$  grid (times  $\tau_j$ )

$$y_{j+1} = (I + A_j \delta t) y_j + b_j \delta t + G_j \delta W_j$$

$$\delta W_j = W(\tau_j + \delta t) - W(\tau_j).$$

Summarizing,

- the oversampling approach yields a simple form of the parameter functionals, which can be solved explicitly on the interval  $[\tau_j, \tau_{j+1}]$ . If  $\delta t$  is small, they are approximately linear in the parameters  $A(t)$ ,  $b(t)$  and  $\Omega(t)$ .
- Thus, also *linear* (in the parameters) KF and SEM software can be used to estimate the system (24).

The linearization of  $\exp(A_j \delta t) = I + A_j \delta t + O(A_j^2 \delta t^2)$  does introduce an approximation error, which must be kept small by choosing a  $\delta t$ , such that  $\|A_j^2\| \delta t^2$  is negligible. In applications  $A_j$  is not known before estimation, so the nonlinear specification (26–28; cf. appendix A) is preferable.

- The price to be paid is the introduction of more latent states  $y_j$  between the measurements. This is no problem in the recursive filtering approach, but the SEM states  $\eta$  and the structural matrices get large (cf. section 4.2 and I).

<sup>3</sup> The intercepts  $b_j = B(\tau_j) \hat{x}(\tau_j) := B_j \hat{x}_j$  are obtained from the interpolated exogenous variables  $\hat{x}(t)$ . A more elaborated nonlinear treatment of (26–28) is given in appendix A.

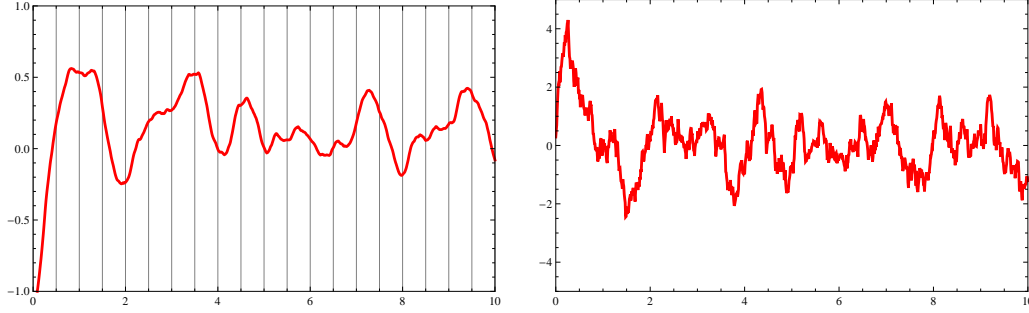


Figure 2: Simulation of the oscillator model with discretization interval  $\delta t = 0.01$  in the interval  $[t_0, t_T] = [0, 10]$ . The sampling interval  $\Delta t = 0.5$  is marked by vertical lines. Also visible is the roughness of the unobserved velocity process (right) which is not differentiable.

- The solution of the difference equation (24) for the indices  $j = j_i, \dots, j_{i+1}$  yields an approximation of the EDM (14) which is exact in the limit  $\delta t \rightarrow 0$ . One obtains the time ordered product representation of the fundamental matrix and approximations of the other integrals automatically and must only choose an interpolation  $\hat{x}(t)$  of  $x(t)$  on the oversampling grid points  $\tau_j$ .

## 4 Applications

### 4.1 Numerical example 1: CAR(2) model

The random oscillator (also called Continuous time AR(2) process, mathematical pendulum, linear oscillator)<sup>4</sup> is defined by the second order differential equation

$$\ddot{y} + \gamma\dot{y} + \omega_0^2 y = bx(t) + g\zeta(t) \quad (32)$$

with the parameters  $\gamma =$  friction,  $\omega_0 = 2\pi/T_o =$  angular frequency,  $T_o =$  period of oscillation,  $g =$  strength of random force  $\zeta$  and exogenous controls  $x(t)$ .

It is obtained from Newton's law

$$\text{Force} = \text{Mass} \times \text{Acceleration} = m \ddot{y}$$

by using a velocity dependent frictional force  $\gamma\dot{y}$  and a linear restoring force  $\omega_0^2 y$ , where  $\omega_0 = 2\pi\nu_0$  is the angular frequency of the undamped oscillation and  $\zeta$  is a random force (white noise) with autocorrelation  $E[\zeta(t)\zeta(s)] = \delta(t - s)$

<sup>4</sup>The model has numerous applications, for example the statistical analysis of sun spots (Bartlett; 1946; Arato et al.; 1962; Singer; 1993), in physics (for an overview see Gitterman; 2005) or in psychology (coupled oscillators, Boker and Poponak; 2004).

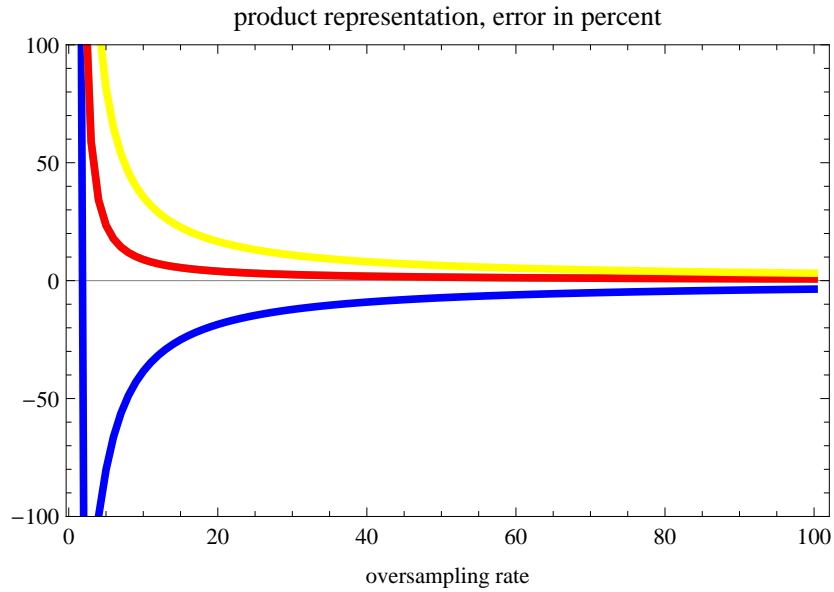


Figure 3: Oversampling of measurement interval  $\Delta t = .5$  with sampling rate  $L = 1, \dots, 100$ . Convergence of the product representation of the matrix exponential function (eqn. 21).

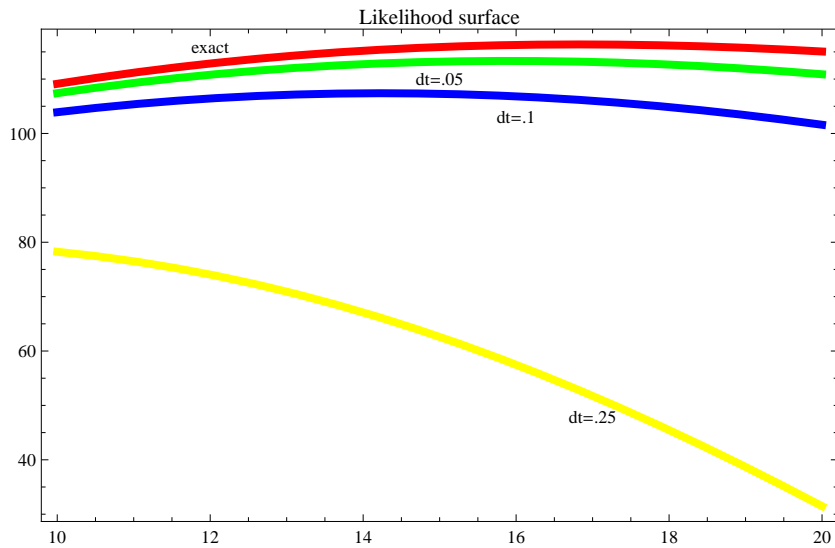


Figure 4: Likelihood surface for the parameter  $\omega_0^2$ . Shown is the exact result and the functions using linearized oversampling  $\delta t = .25, .1, .05$ . The data  $z_i$  were simulated in the interval  $[0, 50]$  using  $\Delta t = 0.5, i = 0, \dots, T = 100$ .

parameters		SEM		KF	
true		$\hat{\theta}$	std	$\hat{\theta}$	std
$\omega_0^2$	16	12.1351	2.00838	12.1351	2.00834
$\gamma$	4	2.83901	0.719584	2.83901	0.719573
$b$	1	0.16749	0.219823	0.16749	0.219824
$g$	2	1.26954	0.26547	1.26954	0.265466
lik		121.341		121.341	

parameters		SEM, lin. oversampling, $J = 5$		KF, lin. oversampling, $J = 5$	
true		$\hat{\theta}$	std	$\hat{\theta}$	std
$\omega_0^2$	16	11.0852	1.72094	11.0852	1.72093
$\gamma$	4	3.63523	0.657319	3.63523	0.657312
$b$	1	0.147298	0.196889	0.147298	0.196888
$g$	2	1.12249	0.20603	1.12249	0.206028
lik		121.509		121.509	

Table 1: CAR(2) model,  $T = 100$ . Comparison of SEM and KF ML-estimates and likelihood. Top: exact EDM ( $\Delta t = .5$ ), bottom: linearized EDM, oversampling ( $J = 5$ ;  $\delta t = .1$ )

(Dirac delta-function).<sup>5</sup> The term  $bx(t)$  represents external forces and can model intercepts. The force terms are scaled by the mass  $m$ .

The pendulum has state space representation

$$d \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} := \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & -\gamma \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} dt + \begin{bmatrix} 0 \\ b \end{bmatrix} x(t) dt + \begin{bmatrix} 0 & 0 \\ 0 & g \end{bmatrix} d \begin{bmatrix} W_1(t) \\ W_2(t) \end{bmatrix} \quad (33)$$

$$z_i := \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} y_1(t_i) \\ y_2(t_i) \end{bmatrix} + \epsilon_i; \quad i = 0, \dots, T. \quad (34)$$

Thus, the SDE of second order can be represented by a vector autoregression at the sampling times  $t_i$ . It is important to note that  $dy_1 = y_2 dt$  or  $dy_1/dt = \dot{y} = y_2$  is an unobserved velocity state which can be reconstructed by the Kalman filter or as smoothed latent vector  $E[y_2(t)|Z^T]$ . *No approximation of derivatives (e.g. numerical differences  $\Delta y_1/\Delta t_i$ ) is necessary.* The velocity  $\dot{y}(t_i) = y_2(t_i)$  at the measurement times  $t_i$  is unobserved (latent).

The eigenvalues of the drift matrix  $A$  are given by  $\lambda_{1,2} = -\gamma/2 \pm i\sqrt{-\gamma^2/4 + \omega_0^2}$   $:= -\gamma/2 \pm i\omega$ , thus the undamped frequency is lowered by the friction.

The stationary values are  $\mu_s = A^{-1}Bx = \begin{bmatrix} bx/\omega_0^2 \\ 0 \end{bmatrix}$  and  $\Sigma_s = (g^2/2\gamma) \begin{bmatrix} 1/\omega_0^2 & 0 \\ 0 & 1 \end{bmatrix}$ ,

which is the solution of  $A\Sigma_s + \Sigma_s A' + \Omega = 0$ . Thus the friction  $\gamma > 0$  is necessary to compensate for the random forces in order to admit a stationary solution (for constant  $x(t) = x$ ).

<sup>5</sup>The formal derivative  $\zeta(t) = dW(t)/dt$  is the white noise process; cf. Arnold (1974).

The EDM has the form

$$\begin{bmatrix} y_{1,i+1} \\ y_{2,i+1} \end{bmatrix} = \begin{bmatrix} a_{11}^* & a_{12}^* \\ a_{21}^* & a_{22}^* \end{bmatrix} \begin{bmatrix} y_{1,i} \\ y_{2,i} \end{bmatrix} + \begin{bmatrix} b_{1,i}^* \\ b_{2,i}^* \end{bmatrix} + \begin{bmatrix} u_{1i} \\ u_{2i} \end{bmatrix}; \quad i = 0, \dots, T-1 \quad (35)$$

$$z_i = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} y_{1,i} \\ y_{2,i} \end{bmatrix} + \epsilon_i; \quad i = 0, \dots, T. \quad (36)$$

In this example only constant controls  $x(t) = 1$  are considered. Thus, the matrices of the EDM are explicitly

$$A_i^* = \exp(A\Delta t_i) \quad (37)$$

$$= \exp(-\gamma/2\Delta t_i) \times \begin{bmatrix} \gamma/(2\omega) \sin \omega \Delta t_i + \cos \omega \Delta t_i & (1/\omega) \sin \omega \Delta t_i \\ -(\omega_0/\omega) \sin \omega \Delta t_i & \cos \omega \Delta t_i - \gamma/(2\omega) \sin \omega \Delta t_i \end{bmatrix}$$

$$B_i^* = A_i^{*-1}(A_i^* - I)B \quad (38)$$

$$\text{row}(\Omega_i^*) = (A \otimes I + I \otimes A)^{-1}(A_i^* \otimes A_i^* - I)\text{row}(\Omega). \quad (39)$$

where

$$A = \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & -\gamma \end{bmatrix}; B = \begin{bmatrix} 0 \\ b \end{bmatrix}; \Omega = \begin{bmatrix} 0 & 0 \\ 0 & g^2 \end{bmatrix}. \quad (40)$$

The true numerical values are set to  $\psi = \{\omega_0^2, \gamma, b, g, \mu_1, \mu_2, \sigma_{11}, \sigma_{12}, \sigma_{22}\} = \{16, 4, 1, 2, 0, 0, 1, 0, 1\}$  where  $\mu = E[y(t_0)]$ ,  $\Sigma = \text{Var}(y(t_0))$  are the parameters of the initial condition.

The matrices of the exact discrete model (37–39) are complicated nonlinear functions of the SDE matrices  $A, B, \Omega$  and the sampling interval  $\Delta t_i$ . In particular, they loose the original restrictions. The numerical values are ( $\Delta t_i = 0.5$ )

$$A_i^* = \begin{bmatrix} 0.150574 & 0.10482 \\ -1.67712 & -0.268705 \end{bmatrix}$$

$$B_i^* = \begin{bmatrix} 0.0530891 \\ 0.10482 \end{bmatrix}$$

$$\Omega_i^* = \begin{bmatrix} 0.0250479 & 0.0219744 \\ 0.0219744 & 0.376001 \end{bmatrix}$$

From this it is easily seen that unrestricted estimation of the \*-matrices does not work, since the restrictions on the SDE matrices (40) cannot be implemented.<sup>6</sup> Fig. 2 shows a part of the simulated trajectory in the interval  $[0, 10]$  with discretization step  $\delta t = 0.01$ . The sampling interval  $\Delta t = 0.5$  is marked by vertical lines. The first component is differentiable ( $\dot{y} = \text{velocity}$ ), but the second derivative (acceleration)  $\ddot{y} = (d/dt)\dot{y}$  does not exist.

<sup>6</sup>The discussion on identification, embedding and aliasing is well known; cf. Singer and Spilerman (1976); Phillips (1976a); Hansen and Sargent (1983); Hamerle et al. (1991, 1993); Singer (1992a).

The simulation in fig. 2 was done with the SEM model (I, 11–12), where the EDM matrices were substituted for the discrete time matrices ( $\alpha_j \rightarrow A_j^*$  etc.). The latent vector  $\eta$  contains the states  $[y_j, \dot{y}_j], j = 0, \dots, J$ , but only the components  $y_{j_i}, i = 0, \dots, T$  are measured. Thus, in the measurement model (I, 12), only the components  $y_{j_i}$  are projected outside on measurements  $z_i$ .

The model can be estimated using the EDM on the measurement times  $t_i = 0, 0.5, 1, 1.5, \dots$  (nonlinear parameters  $A^* = \exp(A\Delta t)$  etc.) or by using the linearized EDM  $A^*(\delta t) \approx I + A\delta t$  etc. on an oversampled grid with  $\delta t = 0.25, 0.1, .05$  etc. Fig. 3 displays the convergence of the product  $(I + A\Delta t/L)^L$  for the oversampling rates  $L = 1, \dots, 100$ . It shows that for the chosen true matrix  $A$  and sampling interval  $\Delta t = 0.5$  at least 20 or more oversampling times must be used to hold the error reasonably small. On the other hand, no (explicit) matrix exponentials must be computed and linear (in the parameters) standard software can be used.

Fig. 4 shows the likelihood surface of parameter  $\omega_0^2$  for several oversampling intervals (linearized EDM matrices) and the true likelihood obtained by the EDM. Note that the same exact result is obtained when nonlinear oversampling is used (cf. eqn. 20)

## 4.2 Numerical example 2: Coupled oscillators with time varying parameters

Coupled CAR(2) processes are interesting models for the oscillatory behavior of individuals which are also in interaction (cf. Boker and Poponak; 2004). A physical picture are pendula which are connected by a spring, where the spring constant parametrizes the interaction. More generally, one could use coupled nonlinear physical pendula, which is equivalent to interacting particles moving in a periodic potential (cf. Gitterman; 2005, and the literature cited therein).<sup>7</sup> Models of this kind are well known in physics, e.g. for diffusing ions in superionic conductors, phonon fields, heat baths etc. In this article, the estimation of the system parameters and filtering and smoothing of the latent states is of central interest.

The random oscillator of example 1 is the basic ingredient. It is coupled to the other pendulum by a force proportional to the distance. More generally, a linear force  $F(y_1, y_2) = \epsilon_1 y_1 + \epsilon_2 y_2 + \epsilon_3$  is introduced which may be time dependent as well. In a physical picture one can set  $F(y_1, y_2) = K(y_1 - y_2 + c)$  with a coupling (spring) constant  $K$  and  $Kc$  is an initial tension.

The coupled oscillators are defined by (cf. 32)

$$\begin{aligned} \ddot{y}_1 + \gamma_1 \dot{y}_1 + \omega_{01}^2 y_1 &= F(y_1, y_2) + g_1 \zeta_1(t) \\ \ddot{y}_2 + \gamma_2 \dot{y}_2 + \omega_{02}^2 y_2 &= -F(y_1, y_2) + g_2 \zeta_2(t), \end{aligned}$$

<sup>7</sup>see also [http://www.fernuni-hagen.de/imperia/md/content/ls\\_statistik/sde.zip](http://www.fernuni-hagen.de/imperia/md/content/ls_statistik/sde.zip) for nonlinear filtering.

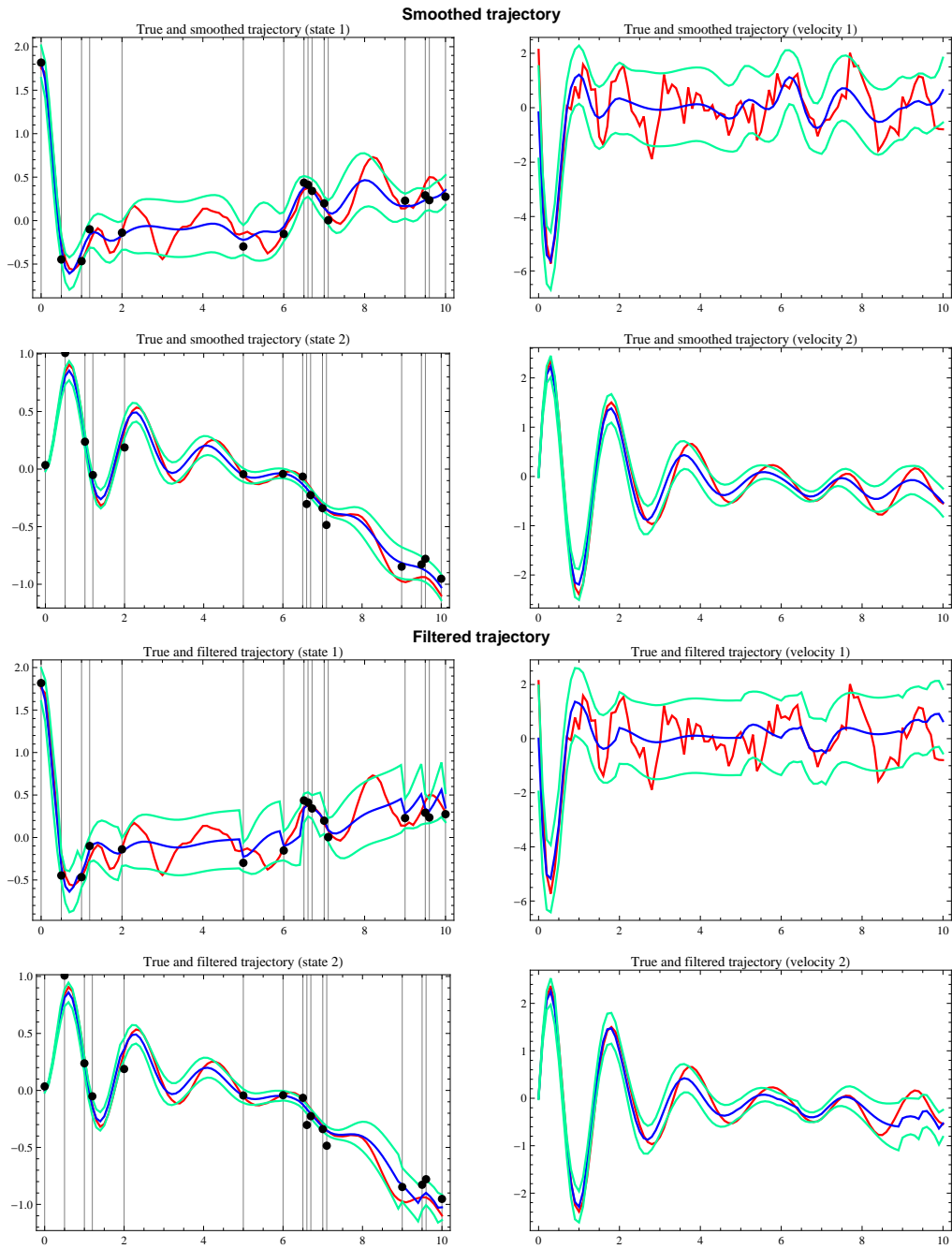


Figure 5: Coupled oscillators: smoothed and filtered trajectories and prediction bands using the true parameter values (see text). The dots represent measurements (error  $\text{Var}(\epsilon_i) = R = 0.01$ ).

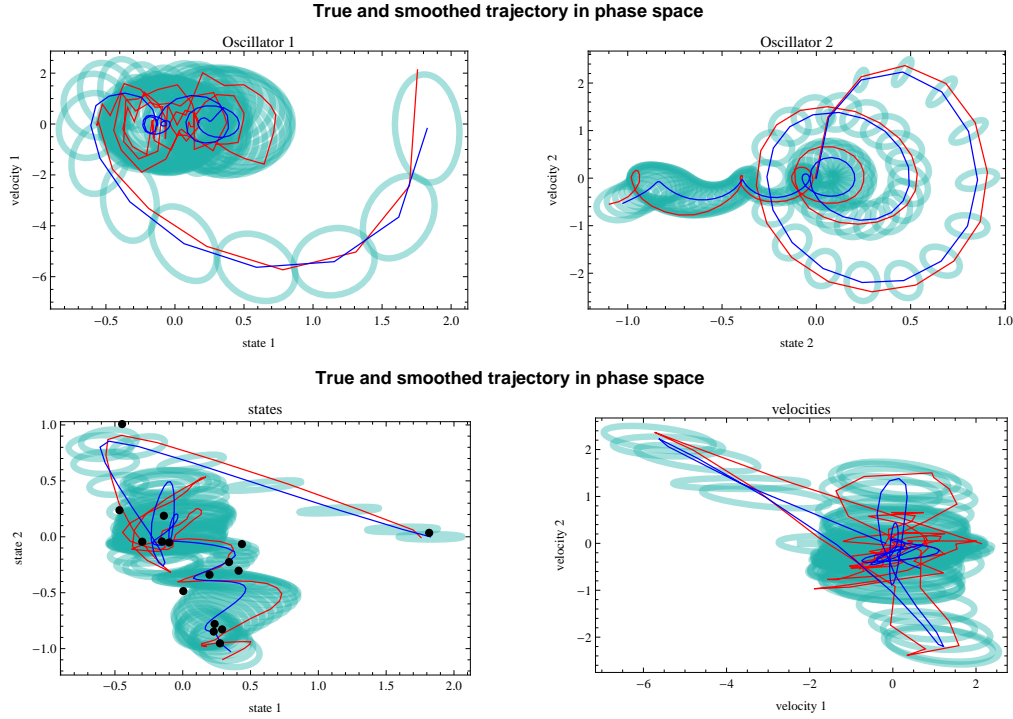


Figure 6: Coupled oscillators: true (red) and smoothed trajectories (blue) with 95% prediction ellipses in 4 dimensional phase space: state vs. velocity projections (top) and state vs. state, velocity vs. velocity (bottom). The dots represent measurements.

but more general specifications are possible (continuous time CVAR(2), including coupling via frictional forces).

For the purpose of filtering and estimation, the state space representation

$$\begin{aligned}
 d/dt \begin{bmatrix} y_1 \\ \dot{y}_1 \\ y_2 \\ \dot{y}_2 \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_{01}^2 + \epsilon_1 & -\gamma_1 & \epsilon_2 & 0 \\ 0 & 0 & 0 & 1 \\ -\epsilon_1 & 0 & -\omega_{02}^2 - \epsilon_2 & -\gamma_2 \end{bmatrix} \begin{bmatrix} y_1 \\ \dot{y}_1 \\ y_2 \\ \dot{y}_2 \end{bmatrix} \\
 &+ \begin{bmatrix} 0 \\ \epsilon_3 \\ 0 \\ -\epsilon_3 \end{bmatrix} [1] + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & g_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & g_2 \end{bmatrix} \begin{bmatrix} \zeta_{11} \\ \zeta_{12} \\ \zeta_{21} \\ \zeta_{21} \end{bmatrix}
 \end{aligned}$$

is used. In order to model a changing interaction, the interaction parameters  $\epsilon_l(t) = \epsilon_{l0} + \epsilon_{l1}t$ ;  $l = 1, 2, 3$  are assumed to be time varying (other functional forms can be used as well). For the simulation of data I used the true parameter values  $\omega_{01}^2 = 16$ ,  $\gamma_1 = 4$ ,  $\epsilon_1(t) = \epsilon_{10} + \epsilon_{11}t = -5 + 1t$ ,  $g_1 = 2$  (first oscillator),  $\omega_{02}^2 = 10$ ,  $\gamma_2 = 0$ ,  $\epsilon_2(t) = -\epsilon_1(t)$ ,  $g_2 = 0$  (second oscillator) and  $\epsilon_3(t) = \epsilon_1(t)$ . Thus the interaction force is  $F(y_1, y_2, t) = (-5+1t)(y_1 - y_2 + 1)$ . For  $t = 0$ , the



		nonlinear oversampling, $\delta t = .1$					
		SEM, inverse		SEM, g-inverse		KF	
parameter	true	$\hat{\psi}$	std	$\hat{\psi}$	std	$\hat{\psi}$	std
$\omega_{01}^2$	16	19.5392	3.9437	19.5392	3.9435	19.5392	3.9437
$\gamma_1$	4	3.627	1.1126	3.627	1.1125	3.627	1.1126
$\epsilon_{10}$	-5	-4.5474	0.6699	-4.5474	0.6699	-4.5474	0.6699
$\epsilon_{11}$	1	0.9007	0.0919	0.9007	0.0919	0.9007	0.0919
$g_1$	2	2.0289	0.6753	2.0289	0.6752	2.0289	0.6752
$\omega_{02}^2$	10	9.4755	1.0699	9.4755	1.0699	9.4755	1.0699
$\gamma_2$	1	1.0069	0.413	1.0069	0.413	1.0069	0.413
$g_2$	0	-0.2132	0.2914	0.2132	0.2914	0.2132	0.2914
time, likelihood		0.659547, 40.6223		0.659343, 40.6223		0.071734, 40.6223	

Table 2: Nonlinear oversampling: ML estimates for the irregularly sampled data (see fig. 5). Comparison of SEM (inverse, g-inverse) with KF. See text.

		nonlinear oversampling					
		$\delta t = .1$		$\delta t = .05$		$\delta t = .025$	
parameter	true	$\hat{\psi}$	std	$\hat{\psi}$	std	$\hat{\psi}$	std
$\omega_{01}^2$	16	19.5392	3.9437	19.5381	3.9433	19.5378	3.9431
$\gamma_1$	4	3.627	1.1126	3.6266	1.1125	3.6265	1.1124
$\epsilon_{10}$	-5	-4.5474	0.6699	-4.5685	0.6716	-4.5794	0.6725
$\epsilon_{11}$	1	0.9007	0.0919	0.9005	0.0919	0.9004	0.0919
$g_1$	2	2.0289	0.6752	2.0287	0.6752	2.0287	0.6752
$\omega_{02}^2$	10	9.4755	1.0699	9.4744	1.0695	9.4742	1.0694
$\gamma_2$	1	1.0069	0.413	1.007	0.4129	1.007	0.4129
$g_2$	0	0.2132	0.2914	0.2131	0.2913	-0.2131	0.2912
time, lik (SEM)		0.668482, 40.6223		2.7434, 40.3894		12.9137, 40.2625	
time, lik (KF)		0.068108, 40.6223		0.128167, 40.3894		0.241293, 40.2625	

Table 3: Nonlinear oversampling: ML estimates for irregularly sampled data. Oversampling times  $\delta t = 0.1, 0.05, 0.025$ . Numbers rounded to 4 digits. The likelihoods and evaluation times are shown in the last two rows.

		linear oversampling					
		$\delta t = .1$		$\delta t = .05$		$\delta t = .025$	
parameter	true	$\hat{\psi}$	std	$\hat{\psi}$	std	$\hat{\psi}$	std
$\omega_{01}^2$	16	19.7074	4.5127	19.7774	3.8744	19.6349	4.0105
$\gamma_1$	4	5.9404	1.6527	4.7587	1.2193	4.1755	1.1857
$\epsilon_{10}$	-5	-4.2604	0.6824	-4.2996	0.522	-4.4444	0.6463
$\epsilon_{11}$	1	0.9093	0.1126	0.8921	0.0766	0.896	0.0838
$g_1$	2	2.277	0.7624	2.1573	0.6786	2.0899	0.6882
$\omega_{02}^2$	10	10.0656	0.8584	9.8408	0.7096	9.6502	1.0415
$\gamma_2$	1	2.4442	0.9375	1.5869	0.3929	1.3007	0.4299
$g_2$	0	0	0.5892	0	0.9681	-0.1462	0.3856
time, lik (SEM)		0.64668, -16.9562		2.70384, 32.2323		12.9647, 39.0207	
time, lik (KF)		0.047711, -16.9562		0.080217, 32.2323		0.153833, 39.0207	

Table 4: Linear oversampling: ML estimates for irregularly sampled data. Oversampling times  $\delta t = 0.1, 0.05, 0.025$ . Numbers rounded to 4 digits. The likelihoods and evaluation times are shown in the last two rows.

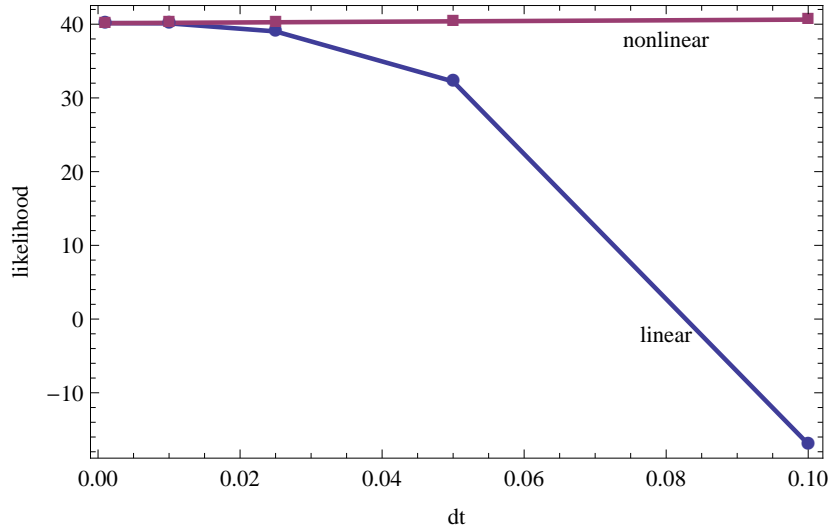


Figure 7: Coupled oscillators: Convergence of likelihoods (linearized and non-linear parameter functionals) as a function of the oversampling intervals  $\delta t = \{.1, .05, .025, .01, .001\}$ .

term  $F(y_1, y_2, 0)$  leads to an attraction of the oscillators, at  $t = 5$  the interaction vanishes and for  $t > 5$  increased repulsion takes part. The first oscillator is the same as in example 2, the second one is purely deterministic without random force and starts at  $y_2 = 0$  at rest ( $\dot{y}_2 = 0$ ). Due to the coupling it starts to move (first it is attracted by 1, but later it is indifferent and then repelled by 1; and vice versa).

Fig. 5 shows the true trajectories (state and velocity) of the oscillators together with approximate 95 % prediction intervals, computed with the true parameters. The filtered states (bottom) were computed with the Kalman filter and show  $E[y(t)|Z^t]_i \pm 1.96\sqrt{\text{Var}[y(t)|Z^t]_{ii}}$ ,  $i = 1, \dots, 4$ . The smoothed state uses all information, i.e.  $E[y(t)|Z^T]$ . The results obtained by the SEM and the KF approach are very similar, although the recursive approach is much faster, both in simulation and filtering.

2-dimensional projections of the 4-dimensional movement in phase space are shown in fig. 6 (red lines). Also displayed is the (projected) smoothed state  $E[(y_1, \dot{y}_1, y_2, \dot{y}_2)(t)|Z^T]$  (blue) and 95% prediction ellipses.

The parameters were estimated with the SEM-ML procedure with linear and nonlinear oversampling. Table 2 compares the results of the SEM (inverse and g-inverse of the covariance matrix  $\Sigma_y$ ) with the Kalman filter. The results are almost identical, but the KF approach is much faster.

Tables 3–4 show the comparison of the ML estimates (nonlinear oversampling) with linearized oversampling ( $\delta t = .1, 0.05, .025$ ). With increasing oversampling rate  $L = 1, 2, 4$ , the linearized ML estimates approach the nonlinear ones. The maximal dimensions  $(J+1)p = (10/.025 + 1)4 = 1604$  of the SEM state  $\eta$  are quite big, however. Therefore, the likelihood evaluation is very slow in the SEM approach. The execution time for one likelihood evaluation is shown in the last lines of the tables.

Fig. 7 shows the convergence of the likelihoods as a function of oversampling interval  $\delta t = \{.1, .05, .025, .01, .001\}$  for the nonlinear and linearized parameter functionals. It should be noted that due to the time dependence of the parameters, the nonlinear oversampling with  $\exp(A(\tau_j)\delta t)$  leads to slightly different results depending on the oversampling interval (cf. appendix A).

## 5 Conclusion

Sampled linear stochastic differential equations can be represented by vector autoregressions, but the parameter functionals are nonlinearly dependent on the original SDE parameters. Both in the SEM and the KF approach, this must be implemented explicitly. Time dependent models can be treated by the oversampling device which corresponds to an Euler approximation of the SDE on a latent grid. Arbitrary interpolation schemes of the exogenous variables are permitted. If the grid spacing is small enough, linearized parameter functionals can be used.

Thus no eigenvalue computations or complex arithmetic is necessary. The price is a large SEM state, but the recursive KF approach still works. Both the SEM and the KF approach are valid for time series and panel data.

## Appendix A: Nonlinear oversampling

The exact parameter functionals

$$\begin{aligned}
A_j^* &= \Phi(\tau_j + \delta t, \tau_j) \\
b_j^* &= \int_{\tau_j}^{\tau_j + \delta t} \Phi(\tau_j + \delta t, s) B(s) x(s) ds \\
\Omega_j^* &= \int_{\tau_j}^{\tau_j + \delta t} \Phi(\tau_j + \delta t, s) \Omega(s) \Phi'(\tau_j + \delta t, s) ds
\end{aligned} \tag{41}$$

of the oversampled model must be computed explicitly in order to perform simulation, ML estimation and filtering/smoothing explicitly.

If the time dependent matrices  $A(t)$ ,  $\Omega(t)$  and vectors  $b(t) = B(t)\hat{x}(t)$  etc. are approximated by step (simple) functions in the interval  $[\tau_j, \tau_j + \delta t]$ , i.e.

$$A(t) \approx \sum_j A_j \chi_{[\tau_j, \tau_j + \delta t)}(t)$$

etc. where

$$\chi_A(t) = \begin{cases} 1 & t \in A \\ 0 & \text{else} \end{cases}$$

(indicator function of  $A$ ), the integrals can be solved explicitly. One obtains

$$\begin{aligned}
A_j^* &\approx \exp(A_j \delta t); \Phi(s, \tau_j) \approx \exp[A_j(s - \tau_j)] \\
b_j^* &\approx A_j^{-1} [\exp(A_j \delta t) - I] b_j \delta t \\
\Omega_j^* &\approx \int_{\tau_j}^{\tau_j + \delta t} \exp[A_j(\delta t + \tau_j - s)] \Omega_j \exp[A_j'(\delta t + \tau_j - s)] ds
\end{aligned}$$

More complicated expressions are obtained by linear interpolation in  $[\tau_j, \tau_j + \delta t]$  (cf. Hamerle et al.; 1993).

The last term can be written explicitly as (cf., e.g. Singer; 1990)

$$\text{row}(\Omega_j^*) \approx [A_j \otimes I + I \otimes A_j]^{-1} [\exp(A_j \delta t) \otimes \exp(A_j \delta t) - I] \text{row}(\Omega_j).$$

In order to avoid misunderstandings, note that the exogenous variables  $x(t)$  are approximated twofold:

First they are interpolated between the measurement times  $t_i$  leading to  $\hat{x}(t)$  (step functions, polygons, splines etc.). Then, they are evaluated at  $\hat{x}_j = \hat{x}(\tau_j)$  and written as step (simple) functions  $\hat{\hat{x}}(t) \approx \sum_j \hat{x}_j \chi_{[\tau_j, \tau_j + \delta t)}(t)$  to admit an evaluation of (41).

## Appendix B: Numerical considerations

All computations were done using Mathematica 7, which is an interpreter language. The Kalman filter approach is implemented in the LSDE and SDE packages, whereas the SEM computations are obtained with the SEM equations of section 2.<sup>8</sup>

The ML estimator was obtained by using a quasi Newton algorithm with BFGS secant updates (Dennis Jr. and Schnabel; 1983) and numerical scores. At the end of the iteration the asymptotic standard errors were computed from the observed Fisher information  $-(\partial^2 l / \partial \psi \psi')(\hat{\psi})$ . In the SDE approach, analytical score functions were implemented also (Singer; 1990, 1993, 1995).

As mentioned, in the LSDE approach, the matrix exponential function and its derivative  $\partial / \partial \psi \exp(A\Delta t)$  was calculated using the spectral representation  $\exp(A\Delta t) = P \exp(\Lambda \Delta t) P^{-1}$  where  $P$  contains the eigenvectors and  $\Lambda$  is the diagonal matrix of (complex) eigenvalues. In rare cases a Jordan canonical form may be necessary, but we assume that all eigenvalues are distinct (cf. Jennrich and Bright; 1976; Singer; 1990).

In the SEM approach, the Mathematica kernel function `MatrixExp` was utilized. According to the online help, it uses variable-order Padé approximation, evaluating rational matrix functions using Paterson-Stockmeyer methods, or Krylov subspace approximations.<sup>9</sup> For a thorough discussion of the matrix exponential and its derivative, see Wilcox (1967); Najfeld and Havel (1995); Moler and VanLoan (2003).

The software permits arbitrary nonlinear matrix restrictions since all matrices are functions of a parameter vector  $\psi$  (e.g.  $\exp(A(\psi)\Delta t)$ ).

In the oversampling approach (Euler approximation), the linearized  $\exp(A\delta t) \approx I + A\delta t$  can be used with error controlled by  $\delta t$ . Thus one could work with linear (in the parameters) programs like LISREL, at least for  $N > 1$  to avoid singularity of the moment matrices (but cf. Hamerle et al.; 1991). The oversampling (better approximation for  $\exp(A\delta t)$ ) leads to larger state dimensions  $(J + 1)p$ ;  $J = (t_T - t_0) / \delta t$ , however.

In the SEM program, the structural matrices (of order  $(J + 1)p \times (J + 1)p$ ) are computed automatically by block matrix operations, which may be somewhat tedious in other systems.

Generally, in my experience, the SEM approach only works satisfactorily (in terms of numerical stability and speed) if  $(J + 1)p \leq 100$ . The KF approach is only limited by the dimensions  $p$  and  $k$  of the state variables.

---

<sup>8</sup>see:

[http://www.fernuni-hagen.de/imperia/md/content/ls\\_statistik/sde.zip](http://www.fernuni-hagen.de/imperia/md/content/ls_statistik/sde.zip),

[http://www.fernuni-hagen.de/imperia/md/content/ls\\_statistik/publikationen/semarchive.exe](http://www.fernuni-hagen.de/imperia/md/content/ls_statistik/publikationen/semarchive.exe)

<sup>9</sup><http://reference.wolfram.com/mathematica/note/SomeNotesOnInternalImplementation.html#14568>).

## References

- Abrikosov, A., Gorkov, L. and Dzyaloshinsky, I. (1963). *Methods of Quantum Field Theory in Statistical Physics*, Dover, New York.
- Arato, M., Kolmogoroff, A. and Sinai, J. (1962). Estimation of the parameters of a complex stationary Gaussian Markov process, *Sov. Math. Dokl.* **3**: 1368–1371.
- Arminger, G. (1986). Linear stochastic differential equation models for panel data with unobserved variables, in N. Tuma (ed.), *Sociological Methodology*, pp. 187–212.
- Arnold, L. (1974). *Stochastic Differential Equations*, John Wiley, New York.
- Bartlett, M. (1946). On the theoretical specification and sampling properties of autocorrelated time-series, *Journal of the Royal Statistical Society (Supplement)* **7**: 27–41.
- Bartlett, M. (1955, 1978). *An Introduction to Stochastic Processes*, Cambridge University Press, Cambridge.
- Bergstrom, A. (1976a). Non Recursive Models as Discrete Approximations to Systems of Stochastic Differential Equations (1966), in A. Bergstrom (ed.), *Statistical Inference in Continuous Time Models*, North Holland, Amsterdam, pp. 15–26.
- Bergstrom, A. (1988). The history of continuous-time econometric models, *Econometric Theory* **4**: 365–383.
- Bergstrom, A. (ed.) (1976b). *Statistical Inference in Continuous Time Models*, North Holland, Amsterdam.
- Black, F. and Scholes, M. (1973). The pricing of options and corporate liabilities, *Journal of Political Economy* **81**: 637–654.
- Boker, S. M. and Poponak, S. (2004). Multivariate Multilevel Models of Dynamical Systems using Latent Differential Equations, in C. van Dijkum et al. (ed.), *Sixth International Conference on Social Science Methodology*, SISWO, CD ROM, ISBN 90-6706-176-x.
- Coleman, J. (1968). The mathematical study of change, in H. Blalock and B. A.B. (eds), *Methodology in Social Research*, McGraw-Hill, New York, pp. 428–478.
- Dennis Jr., J. and Schnabel, R. (1983). *Numerical Methods for Unconstrained Optimization and Nonlinear Equations*, Prentice Hall, Englewood Cliffs.

- Doreian, P. and Hummon, N. (1976). *Modelling Social Processes*, Elsevier, New York, Oxford, Amsterdam.
- Doreian, P. and Hummon, N. (1979). Estimating differential equation models on time series: Some simulation evidence, *Sociological Methods and Research* **8**: 3–33.
- Gitterman, M. (2005). *The Noisy Oscillator*, World Scientific, Singapore.
- Hamerle, A., Nagl, W. and Singer, H. (1991). Problems with the estimation of stochastic differential equations using structural equations models, *Journal of Mathematical Sociology* **16**, **3**: 201–220.
- Hamerle, A., Nagl, W. and Singer, H. (1993). Identification and estimation of continuous time dynamic systems with exogenous variables using panel data, *Econometric Theory* **9**: 283–295.
- Hansen, L. and Sargent, T. (1983). The dimensionality of the aliasing problem in models with rational spectral density, *Econometrica* **51**: 377–387.
- Itô, K. (1951). On stochastic differential equations, *Memoirs Amer. Math. Soc* **4**: 4.
- Jazwinski, A. (1970). *Stochastic Processes and Filtering Theory*, Academic Press, New York.
- Jennrich, R. and Bright, P. (1976). Fitting Systems of Linear Differential Equations Using Computer Generated Exact Derivatives, *Technometrics* **18**, **4**: 385–392.
- Jones, R. (1984). Fitting multivariate models to unequally spaced data, in E. Parzen (ed.), *Time Series Analysis of Irregularly Observed Data*, Springer, New York, pp. 158–188.
- Jones, R. (1993). *Longitudinal Data With Serial Correlation: A State Space Approach*, Chapman and Hall, New York.
- Kalman, R. and Bucy, R. (1961). New Results in Linear Filtering and Prediction Theory, *Trans. ASME, Ser. D: J. Basic Eng.* **83**: 95–108.
- Langevin, P. (1908). Sur la théorie du mouvement brownien, *C.R. Acad. Sci. Paris* **146**: 530–533.
- Mardia, K., Kent, J. and Bibby, J. (1979). *Multivariate Analysis*, Academic Press, London.
- McDonald, R. and Swaminathan, H. (1973). A simple matrix calculus with applications to multivariate statistics, *General Systems* **XVIII**: 37–54.

- Möbus, C. and Nagl, W. (1983). Messung, Analyse und Prognose von Veränderungen (Measurement, analysis and prediction of change; in german), *Hypothesenprüfung, Band 5 der Serie Forschungsmethoden der Psychologie der Enzyklopädie der Psychologie*, Hogrefe, pp. 239–470.
- Moler, C. and VanLoan, C. (2003). Nineteen Dubious Ways to Compute the Exponential of a Matrix, Twenty-Five Years Later, *SIAM Review* **45,1**: 1–46.
- Najfeld, I. and Havel, T. (1995). Derivatives of the matrix exponential and their computation, *Adv. Appl. Math.* **16**: 321–375.
- Nelson, E. (1967). *Dynamical Theories of Brownian Motion*, Princeton University Press, Princeton.
- Oud, H. and Singer, H. (2008a). Continuous time modeling of panel data: SEM vs. filter techniques, *Statistica Neerlandica* **62,1**: 4–28.
- Oud, H. and Singer, H. (2008b). Special issue: Continuous time modeling of panel data. Editorial introduction, *Statistica Neerlandica* **62,1**: 1–3.
- Oud, J. and Jansen, R. (2000). Continuous Time State Space Modeling of Panel Data by Means of SEM, *Psychometrika* **65**: 199–215.
- Phillips, P. (1976). The estimation of linear stochastic differential equations with exogenous variables, in A. Bergstrom (ed.), *Statistical Inference in Continuous Time Models*, North Holland, Amsterdam, pp. 135–173.
- Phillips, P. (1976a). The problem of identification in finite parameter continuous time models, in A. Bergstrom (ed.), *Statistical Inference in Continuous Time Models*, North Holland, Amsterdam, pp. 123–134.
- Schweppe, F. (1965). Evaluation of likelihood functions for gaussian signals, *IEEE Transactions on Information Theory* **11**: 61–70.
- Singer, B. and Spilerman, S. (1976). Representation of social processes by Markov models, *American Journal of Sociology* **82**: 1–53.
- Singer, H. (1986). *Depressivität und gelernte Hilflosigkeit als Stochastischer Prozeß* [Depression and learned helplessness as stochastic process; in german], Master's thesis, Universität Konstanz, Konstanz, Germany.
- Singer, H. (1990). *Parameterschätzung in zeitkontinuierlichen dynamischen Systemen* [Parameter estimation in continuous time dynamical systems; Ph.D. thesis, in german], Hartung-Gorre-Verlag, Konstanz.
- Singer, H. (1991). *LSDE - A program package for the simulation, graphical display, optimal filtering and maximum likelihood estimation of Linear Stochastic Differential Equations, User's guide*, Meersburg.



- Singer, H. (1992a). The aliasing phenomenon in visual terms, *Journal of Mathematical Sociology* **17**, **1**: 39–49.
- Singer, H. (1992b). *Zeitkontinuierliche Dynamische Systeme*, Campus-Verlag, Frankfurt a.M./New York. [Continuous Time Dynamical Systems; in German].
- Singer, H. (1993). Continuous-time dynamical systems with sampled data, errors of measurement and unobserved components, *Journal of Time Series Analysis* **14**, **5**: 527–545.
- Singer, H. (1995). Analytical score function for irregularly sampled continuous time stochastic processes with control variables and missing values, *Econometric Theory* **11**: 721–735.
- Singer, H. (1998). Continuous Panel Models with Time Dependent Parameters, *Journal of Mathematical Sociology* **23**: 77–98.
- Singer, H. (2007). Stochastic Differential Equation Models with Sampled Data, in K. van Montfort, H. Oud and A. Satorra (eds), *Longitudinal Models in the Behavioral and Related Sciences*, The European Association of Methodology (EAM) Methodology and Statistics series, vol. II, Lawrence Erlbaum Associates, Mahwah, London, pp. 73–106.
- Singer, H. (2009). SEM modeling with singular moment matrices. Part I: ML-Estimation of time series., *Diskussionsbeiträge Fakultät Wirtschaftswissenschaft Nr. 441*, FernUniversität in Hagen. <http://www.fernuni-hagen.de/FBWIFI/forschung/beitraege/pdf/db441.pdf>.
- Stratonovich, R. (1960). Conditional Markov Processes, *Theor. Probability Appl.* **5**: 156–178.
- Uhlenbeck, G. and Ornstein, L. (1954). On the Theory of Brownian Motion (1930), in N. Wax (ed.), *Selected Papers on Noise and Stochastic Processes*, Dover, New York, pp. 93–111.
- Wilcox, R. (1967). Exponential operators and parameter differentiation in quantum physics, *Journal of Mathematical Physics* **8**: 962.
- Zadrozny, P. (1988). Gaussian likelihood of continuous-time armax models when data are stocks and flows at different frequencies, *Econometric Theory* **4**: 108–124.

Die Diskussionspapiere ab Nr. 183 (1992) bis heute, können Sie im Internet unter <http://www.fernuni-hagen.de/FBWIWI/> einsehen und zum Teil downloaden.

Die **Titel** der Diskussionspapiere von Nr 1 (1975) bis 182 (1991) können bei Bedarf in der Fakultät für Wirtschaftswissenschaft angefordert werden:

FernUniversität, z. Hd. Frau Huber oder Frau Mette, Postfach 940, 58084 Hagen

**Die Diskussionspapiere selber erhalten Sie nur in den Bibliotheken.**

Nr	Jahr	Titel	Autor/en
322	2001	Spreading Currency Crises: The Role of Economic Interdependence	Berger, Wolfram Wagner, Helmut
323	2002	Planung des Fahrzeugumschlags in einem Seehafen-Automobilterminal mittels eines Multi-Agenten-Systems	Fischer, Torsten Gehring, Hermann
324	2002	A parallel tabu search algorithm for solving the container loading problem	Bortfeldt, Andreas Gehring, Hermann Mack, Daniel
325	2002	Die Wahrheit entscheidungstheoretischer Maximen zur Lösung von Individualkonflikten - Unsicherheitssituationen -	Mus, Gerold
326	2002	Zur Abbildungsgenauigkeit des Gini-Koeffizienten bei relativer wirtschaftlicher Konzentration	Steinrücke, Martin
327	2002	Entscheidungsunterstützung bilateraler Verhandlungen über Auftragsproduktionen - eine Analyse aus Anbietersicht	Steinrücke, Martin
328	2002	Die Relevanz von Marktzinssätzen für die Investitionsbeurteilung – zugleich eine Einordnung der Diskussion um die Marktzinsmethode	Terstege, Udo
329	2002	Evaluating representatives, parliament-like, and cabinet-like representative bodies with application to German parliament elections 2002	Tangian, Andranik S.
330	2002	Konzernabschluss und Ausschüttungsregelung im Konzern. Ein Beitrag zur Frage der Eignung des Konzernabschlusses als Ausschüttungsbemessungsinstrument	Hinz, Michael
331	2002	Theoretische Grundlagen der Gründungsfinanzierung	Bitz, Michael
332	2003	Historical background of the mathematical theory of democracy	Tangian, Andranik S.
333	2003	MCDM-applications of the mathematical theory of democracy: choosing travel destinations, preventing traffic jams, and predicting stock exchange trends	Tangian, Andranik S.
334	2003	Sprachregelungen für Kundenkontaktmitarbeiter – Möglichkeiten und Grenzen	Fließ, Sabine Möller, Sabine Momma, Sabine Beate

335	2003	A Non-cooperative Foundation of Core-Stability in Positive Externality NTU-Coalition Games	Finus, Michael Rundshagen, Bianca
336	2003	Combinatorial and Probabilistic Investigation of Arrow's dictator	Tangian, Andranik
337	2003	A Grouping Genetic Algorithm for the Pickup and Delivery Problem with Time Windows	Pankratz, Giselher
338	2003	Planen, Lernen, Optimieren: Beiträge zu Logistik und E-Learning. Festschrift zum 60 Geburtstag von Hermann Gehring	Bortfeldt, Andreas Fischer, Torsten Homberger, Jörg Pankratz, Giselher Strangmeier, Reinhard
339a	2003	Erinnerung und Abruf aus dem Gedächtnis Ein informationstheoretisches Modell kognitiver Prozesse	Rödder, Wilhelm Kuhlmann, Friedhelm
339b	2003	Zweck und Inhalt des Jahresabschlusses nach HGB, IAS/IFRS und US-GAAP	Hinz, Michael
340	2003	Voraussetzungen, Alternativen und Interpretationen einer zielkonformen Transformation von Periodenerfolgsrechnungen – ein Diskussionsbeitrag zum LÜCKE-Theorem	Terstege, Udo
341	2003	Equalizing regional unemployment indices in West and East Germany	Tangian, Andranik
342	2003	Coalition Formation in a Global Warming Game: How the Design of Protocols Affects the Success of Environmental Treaty-Making	Eyckmans, Johan Finus, Michael
343	2003	Stability of Climate Coalitions in a Cartel Formation Game	Finus, Michael van Ierland, Ekko Dellink, Rob
344	2003	The Effect of Membership Rules and Voting Schemes on the Success of International Climate Agreements	Finus, Michael J.-C., Altamirano-Cabrera van Ierland, Ekko
345	2003	Equalizing structural disproportions between East and West German labour market regions	Tangian, Andranik
346	2003	Auf dem Prüfstand: Die geldpolitische Strategie der EZB	Kißmer, Friedrich Wagner, Helmut
347	2003	Globalization and Financial Instability: Challenges for Exchange Rate and Monetary Policy	Wagner, Helmut

348	2003	Anreizsystem Frauenförderung – Informationssystem Gleichstellung am Fachbereich Wirtschaftswissenschaft der FernUniversität in Hagen	Fließ, Sabine Nonnenmacher, Dirk
349	2003	Legitimation und Controller	Pietsch, Gotthard Scher, Ewald
350	2003	Controlling im Stadtmarketing – Ergebnisse einer Primärerhebung zum Hagener Schaufenster-Wettbewerb	Fließ, Sabine Nonnenmacher, Dirk
351	2003	Zweiseitige kombinatorische Auktionen in elektronischen Transportmärkten – Potenziale und Probleme	Pankratz, Giselher
352	2003	Methodisierung und E-Learning	Strangmeier, Reinhard Bankwitz, Johannes
353 a	2003	A parallel hybrid local search algorithm for the container loading problem	Mack, Daniel Bortfeldt, Andreas Gehring, Hermann
353 b	2004	Übernahmeangebote und sonstige öffentliche Angebote zum Erwerb von Aktien – Ausgestaltungsmöglichkeiten und deren Beschränkung durch das Wertpapiererwerbs- und Übernahmegesetz	Wirtz, Harald
354	2004	Open Source, Netzeffekte und Standardisierung	Maaß, Christian Scher, Ewald
355	2004	Modesty Pays: Sometimes!	Finus, Michael
356	2004	Nachhaltigkeit und Biodiversität	Endres, Alfred Bertram, Regina
357	2004	Eine Heuristik für das dreidimensionale Strip-Packing-Problem	Bortfeldt, Andreas Mack, Daniel
358	2004	Netzwerkökonomik	Martiensen, Jörn
359	2004	Competitive versus cooperative Federalism: Is a fiscal equalization scheme necessary from an allocative point of view?	Arnold, Volker
360	2004	Gefangenendilemma bei Übernahmeangeboten? Eine entscheidungs- und spieltheoretische Analyse unter Einbeziehung der verlängerten Annahmefrist gem. § 16 Abs. 2 WpÜG	Wirtz, Harald
361	2004	Dynamic Planning of Pickup and Delivery Operations by means of Genetic Algorithms	Pankratz, Giselher

362	2004	Möglichkeiten der Integration eines Zeitmanagements in das Blueprinting von Dienstleistungsprozessen	Fließ, Sabine Lasshof, Britta Meckel, Monika
363	2004	Controlling im Stadtmarketing - Eine Analyse des Hagener Schaufensterwettbewerbs 2003	Fließ, Sabine Wittko, Ole
364	2004	Ein Tabu Search-Verfahren zur Lösung des Timetabling-Problems an deutschen Grundschulen	Desef, Thorsten Bortfeldt, Andreas Gehring, Hermann
365	2004	Die Bedeutung von Informationen, Garantien und Reputation bei integrativer Leistungserstellung	Prechtl, Anja Völker-Albert, Jan- Hendrik
366	2004	The Influence of Control Systems on Innovation: An empirical Investigation	Littkemann, Jörn Derfuß, Klaus
367	2004	Permit Trading and Stability of International Climate Agreements	Altamirano-Cabrera, Juan-Carlos Finus, Michael
368	2004	Zeitdiskrete vs. zeitstetige Modellierung von Preismechanismen zur Regulierung von Angebots- und Nachfragemengen	Mazzoni, Thomas
369	2004	Marktversagen auf dem Softwaremarkt? Zur Förderung der quelloffenen Softwareentwicklung	Christian Maaß Ewald Scherm
370	2004	Die Konzentration der Forschung als Weg in die Sackgasse? Neo-Institutionalistische Überlegungen zu 10 Jahren Anreizsystemforschung in der deutschsprachigen Betriebswirtschaftslehre	Süß, Stefan Muth, Insa
371	2004	Economic Analysis of Cross-Border Legal Uncertainty: the Example of the European Union	Wagner, Helmut
372	2004	Pension Reforms in the New EU Member States	Wagner, Helmut
373	2005	Die Bundestrainer-Scorecard Zur Anwendbarkeit des Balanced Scorecard Konzepts in nicht-ökonomischen Fragestellungen	Eisenberg, David Schulte, Klaus
374	2005	Monetary Policy and Asset Prices: More Bad News for „Benign Neglect“	Berger, Wolfram Kißmer, Friedrich Wagner, Helmut
375	2005	Zeitstetige Modellbildung am Beispiel einer volkswirtschaftlichen Produktionsstruktur	Mazzoni, Thomas

376	2005	Economic Valuation of the Environment	Endres, Alfred
377	2005	Netzwerkökonomik – Eine vernachlässigte theoretische Perspektive in der Strategie-/Marketingforschung?	Maaß, Christian Scher, Ewald
378	2005	Diversity management`s diffusion and design: a study of German DAX-companies and Top-50-U.S.-companies in Germany	Süß, Stefan Kleiner, Markus
379	2005	Fiscal Issues in the New EU Member Countries – Prospects and Challenges	Wagner, Helmut
380	2005	Mobile Learning – Modetrend oder wesentlicher Bestandteil lebenslangen Lernens?	Kuszpa, Maciej Scher, Ewald
381	2005	Zur Berücksichtigung von Unsicherheit in der Theorie der Zentralbankpolitik	Wagner, Helmut
382	2006	Effort, Trade, and Unemployment	Altenburg, Lutz Brenken, Anke
383	2006	Do Abatement Quotas Lead to More Successful Climate Coalitions?	Altamirano-Cabrera, Juan-Carlos Finus, Michael Dellink, Rob
384	2006	Continuous-Discrete Unscented Kalman Filtering	Singer, Hermann
385	2006	Informationsbewertung im Spannungsfeld zwischen der Informationstheorie und der Betriebswirtschaftslehre	Reucher, Elmar
386	2006	The Rate Structure Pattern: An Analysis Pattern for the Flexible Parameterization of Charges, Fees and Prices	Pleiß, Volker Pankratz, Giselher Bortfeldt, Andreas
387a	2006	On the Relevance of Technical Inefficiencies	Fandel, Günter Lorth, Michael
387b	2006	Open Source und Wettbewerbsstrategie - Theoretische Fundierung und Gestaltung	Maaß, Christian
388	2006	Induktives Lernen bei unvollständigen Daten unter Wahrung des Entropieprinzips	Rödter, Wilhelm
389	2006	Banken als Einrichtungen zur Risikotransformation	Bitz, Michael
390	2006	Kapitalerhöhungen börsennotierter Gesellschaften ohne börslichen Bezugsrechtshandel	Terstege, Udo Stark, Gunnar
391	2006	Generalized Gauss-Hermite Filtering	Singer, Hermann

392	2006	Das Göteborg Protokoll zur Bekämpfung grenzüberschreitender Luftschadstoffe in Europa: Eine ökonomische und spieltheoretische Evaluierung	Ansel, Wolfgang Finus, Michael
393	2006	Why do monetary policymakers lean with the wind during asset price booms?	Berger, Wolfram Kißner, Friedrich
394	2006	On Supply Functions of Multi-product Firms with Linear Technologies	Steinrücke, Martin
395	2006	Ein Überblick zur Theorie der Produktionsplanung	Steinrücke, Martin
396	2006	Parallel greedy algorithms for packing unequal circles into a strip or a rectangle	Timo Kubach, Bortfeldt, Andreas Gehring, Hermann
397	2006	C&P Software for a cutting problem of a German wood panel manufacturer – a case study	Papke, Tracy Bortfeldt, Andreas Gehring, Hermann
398	2006	Nonlinear Continuous Time Modeling Approaches in Panel Research	Singer, Hermann
399	2006	Auftragsterminierung und Materialflussplanung bei Werkstattfertigung	Steinrücke, Martin
400	2006	Import-Penetration und der Kollaps der Phillips-Kurve	Mazzoni, Thomas
401	2006	Bayesian Estimation of Volatility with Moment-Based Nonlinear Stochastic Filters	Grothe, Oliver Singer, Hermann
402	2006	Generalized Gauss-Hermite Filtering for Multivariate Diffusion Processes	Singer, Hermann
403	2007	A Note on Nash Equilibrium in Soccer	Sonnabend, Hendrik Schlepütz, Volker
404	2007	Der Einfluss von Schaufenstern auf die Erwartungen der Konsumenten - eine explorative Studie	Fließ, Sabine Kudermann, Sarah Trell, Esther
405	2007	Die psychologische Beziehung zwischen Unternehmen und freien Mitarbeitern: Eine empirische Untersuchung des Commitments und der arbeitsbezogenen Erwartungen von IT-Freelancern	Süß, Stefan
406	2007	An Alternative Derivation of the Black-Scholes Formula	Zucker, Max Singer, Hermann
407	2007	Computational Aspects of Continuous-Discrete Extended Kalman-Filtering	Mazzoni, Thomas
408	2007	Web 2.0 als Mythos, Symbol und Erwartung	Maaß, Christian Pietsch, Gotthard
409	2007	„Beyond Balanced Growth“: Some Further Results	Stijepic, Denis Wagner, Helmut

410	2007	Herausforderungen der Globalisierung für die Entwicklungsländer: Unsicherheit und geldpolitisches Risikomanagement	Wagner, Helmut
411	2007	Graphical Analysis in the New Neoclassical Synthesis	Giese, Guido Wagner, Helmut
412	2007	Monetary Policy and Asset Prices: The Impact of Globalization on Monetary Policy Trade-Offs	Berger, Wolfram Kißner, Friedrich Knütter, Rolf
413	2007	Entropiebasiertes Data Mining im Produktdesign	Rudolph, Sandra Röder, Wilhelm
414	2007	Game Theoretic Research on the Design of International Environmental Agreements: Insights, Critical Remarks and Future Challenges	Finus, Michael
415	2007	Qualitätsmanagement in Unternehmenskooperationen - Steuerungsmöglichkeiten und Datenintegrationsprobleme	Meschke, Martina
416	2007	Modernisierung im Bund: Akteursanalyse hat Vorrang	Pietsch, Gotthard Jamin, Leander
417	2007	Inducing Technical Change by Standard Oriented Environmental Policy: The Role of Information	Endres, Alfred Bertram, Regina Rundshagen, Bianca
418	2007	Der Einfluss des Kontextes auf die Phasen einer SAP-Systemimplementierung	Littkemann, Jörn Eisenberg, David Kuboth, Meike
419	2007	Endogenous in Uncertainty and optimal Monetary Policy	Giese, Guido Wagner, Helmut
420	2008	Stockkeeping and controlling under game theoretic aspects	Fandel, Günter Trockel, Jan
421	2008	On Overdissipation of Rents in Contests with Endogenous Intrinsic Motivation	Schlepütz, Volker
422	2008	Maximum Entropy Inference for Mixed Continuous-Discrete Variables	Singer, Hermann
423	2008	Eine Heuristik für das mehrdimensionale Bin Packing Problem	Mack, Daniel Bortfeldt, Andreas
424	2008	Expected A Posteriori Estimation in Financial Applications	Mazzoni, Thomas
425	2008	A Genetic Algorithm for the Two-Dimensional Knapsack Problem with Rectangular Pieces	Bortfeldt, Andreas Winter, Tobias
426	2008	A Tree Search Algorithm for Solving the Container Loading Problem	Fanslau, Tobias Bortfeldt, Andreas
427	2008	Dynamic Effects of Offshoring	Stijepic, Denis Wagner, Helmut
428	2008	Der Einfluss von Kostenabweichungen auf das Nash-Gleichgewicht in einem nicht-kooperativen Disponenten-Controller-Spiel	Fandel, Günter Trockel, Jan
429	2008	Fast Analytic Option Valuation with GARCH	Mazzoni, Thomas
430	2008	Conditional Gauss-Hermite Filtering with Application to Volatility Estimation	Singer, Hermann
431	2008	Web 2.0 auf dem Prüfstand: Zur Bewertung von Internet-Unternehmen	Christian Maaß Gotthard Pietsch



432	2008	Zentralbank-Kommunikation und Finanzstabilität – Eine Bestandsaufnahme	Knütter, Rolf Mohr, Benjamin
433	2008	Globalization and Asset Prices: Which Trade-Offs Do Central Banks Face in Small Open Economies?	Knütter, Rolf Wagner, Helmut
434	2008	International Policy Coordination and Simple Monetary Policy Rules	Berger, Wolfram Wagner, Helmut
435	2009	Matchingprozesse auf beruflichen Teilarbeitsmärkten	Stops, Michael Mazzoni, Thomas
436	2009	Wayfindingprozesse in Parksituationen - eine empirische Analyse	Fließ, Sabine Tetzner, Stefan
437	2009	ENTROPY-DRIVEN PORTFOLIO SELECTION a downside and upside risk framework	Rödder, Wilhelm Gartner, Ivan Ricardo Rudolph, Sandra
438	2009	Consulting Incentives in Contests	Schlepütz, Volker
439	2009	A Genetic Algorithm for a Bi-Objective Winner-Determination Problem in a Transportation-Procurement Auction"	Buer, Tobias Pankratz, Giselher
440	2009	Parallel greedy algorithms for packing unequal spheres into a cuboidal strip or a cuboid	Kubach, Timo Bortfeldt, Andreas Tilli, Thomas Gehring, Hermann
441	2009	SEM modeling with singular moment matrices Part I: ML-Estimation of time series	Singer, Hermann
442	2009	SEM modeling with singular moment matrices Part II: ML-Estimation of sampled stochastic differential equations	Singer, Hermann